

METHODICAL INSTRUCTIONS

to practical classes

on discipline

«ENGINEERING ELECTROMAGNETICS»

Module 1 – Theoretical Fundamentals of Electromagnetics

for students of full-time and part-time forms of education
on telecommunications training area

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Goal of the methodical instructions to practical classes is to help students during independent study of theoretical principles of discipline "Engineering Electrodynamics", part 1 – theoretical foundations of electromagnetics and in the performance of individual tasks. Methodical recommendations to practical classes on the following topics: introduction to electromagnetism, basics of transmission lines are given in the textbook. Key questions on the corresponding topics are given for each lesson as well as problems for the home task followed by solution guidelines.

Methodical instructions to practical classes are intended for students of full-time and part-time forms of education on telecommunications training area.

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1 INTRODUCTION TO ELECTROMAGNETISM. BASIC EQUATIONS OF ELECTROMAGNETICS

1.1 Mathematical tools of electromagnetics: complex numbers, basics of vector analysis

1.1.1 Goal of the practical training

Recurrence of mathematical tools used in an electromagnetics;
development of problems solution skills .

1.1.2 Key points

A *complex number* is an expression of the following kind: $\dot{a} = a' + ia''$, where a' and a'' are real numbers; i is the imaginary unit determined from the expressions

$$ii \equiv i^2 = -1, \quad i = e^{i\frac{\pi}{2}}.$$

Here and after the sign $\langle \equiv \rangle$ specifies transition to other label.

Two complex numbers $a' + ia''$ and $a' - ia''$ are called *complex conjugate* numbers. The complex conjugate number is designated by a «*» sign, i.e. numbers \dot{a} and \dot{a}^* are complex conjugate numbers.

Consider a complex plane (fig. 1.1).

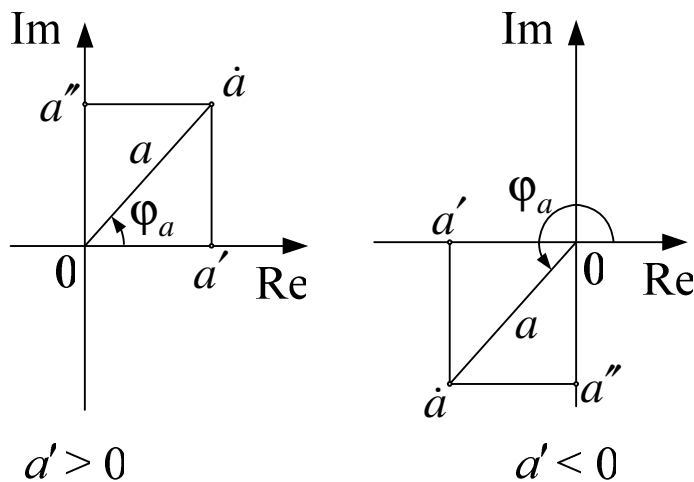


Figure 1.1 – Complex number on a complex plane

Each point of a complex plane corresponds to a complex number \dot{a} which is possible to be presented in the algebraic form:

$$\dot{a} = a' + ia'', \quad (1.1)$$

or the exponential form:

$$\dot{a} = ae^{i\varphi_a}. \quad (1.2)$$

where a' and a'' are real $\text{Re}(\dot{a})$ and imaginary $\text{Im}(\dot{a})$ parts of complex number \dot{a} ;

a and φ_a are module and argument (phase) of the complex number.

Sometimes the number a' is called an **abscissa** ($\text{Re}(\dot{a})$ – real part), and a'' is called **ordinate** ($\text{Im}(\dot{a})$ – imaginary part) of the complex number $\dot{a} = a' + ia''$.

From fig. 1.1 and expressions (1.1), (1.2) one may derive:

$$a = \sqrt{a'^2 + a''^2}, \quad (1.3)$$

$$\begin{cases} \varphi_a = \tan^{-1} \frac{a''}{a'}, & \text{where } a' > 0 \\ \varphi_a = \pi + \tan^{-1} \frac{a''}{a'}, & \text{where } a' < 0 \end{cases} \quad (1.4)$$

$$a' = a \cos \varphi_a, \quad a'' = a \sin \varphi_a. \quad (1.5)$$

Addition, subtraction, multiplication and division of complex numbers $\dot{a} = a' + ia''$ and $\dot{b} = b' + ib''$ is performed with the help of the expressions:

$$\dot{a} \pm \dot{b} = (a' \pm b') + i(a'' \pm b''), \quad (1.6)$$

$$\dot{a}\dot{b} = (a'b' - a''b'') + i(a'b'' + a''b') = abe^{i(\varphi_a + \varphi_b)}, \quad (1.7)$$

$$\frac{\dot{a}}{\dot{b}} = \frac{a' + ia''}{b' + ib''} = \frac{a'b' + a''b''}{b'^2 + b''^2} + i \frac{b'a'' - a'b''}{b'^2 + b''^2} = \frac{a}{b} e^{i(\varphi_a - \varphi_b)}, \quad (1.8)$$

where φ_a, φ_b are accordingly arguments (phases) of complex numbers \dot{a} and \dot{b} .

At the heart of the **vector analysis** the concept **of a vector** (a directional line) lies. A vector is possible to be presented in a general (not coordinate) view as $\vec{A} = \vec{a}_0 A$, where \vec{a}_0 is an ort (unit vector), co-directed with a vector \vec{A} ; A is a module (length) of a vector \vec{A} , and also in the form of the sum of three perpendicular vectors, co-directed with the orts of the given coordinate system.

In the Cartesian, cylindrical and spherical coordinate systems vector \vec{A} representation via the orts looks like the following

$$\vec{A} = \vec{x}_0 A_x + \vec{y}_0 A_y + \vec{z}_0 A_z, \quad (1.9)$$

where $\vec{x}_0, \vec{y}_0, \vec{z}_0$ are the orts of Cartesian coordinates (fig. 1.2); A_x, A_y, A_z are projections of a vector \vec{A} on corresponding directions of the Cartesian coordinate system

$$\vec{A} = \vec{r}_0 A_r + \vec{\Phi}_0 A_\Phi + \vec{z}_0 A_z; \quad (1.10)$$

where $\vec{r}_0, \vec{\Phi}_0, \vec{z}_0$ are the orts of cylindrical coordinates (fig. 1.3a);

A_r, A_Φ, A_z are projections of a vector \vec{A} on corresponding directions of the cylindrical coordinate system

$$\vec{A} = \vec{r}_0 A_r + \vec{\theta}_0 A_\theta + \vec{\Phi}_0 A_\Phi, \quad (1.11)$$

where $\vec{r}_0, \vec{\theta}_0, \vec{\Phi}_0$ are the orts of spherical coordinates (fig. 1.3b);

A_r, A_θ, A_Φ are projections of a vector \vec{A} on corresponding directions of the spherical coordinate system.

If vectors \vec{A} and \vec{B} are presented via its projections, for example, in the Cartesian coordinates:

$$\vec{A} = \vec{x}_0 A_x + \vec{y}_0 A_y + \vec{z}_0 A_z; \quad \vec{B} = \vec{x}_0 B_x + \vec{y}_0 B_y + \vec{z}_0 B_z, \quad (1.12)$$

the projections of the sum (difference) of vectors \vec{A} and \vec{B} are equal to the sums (differences) of corresponding projections

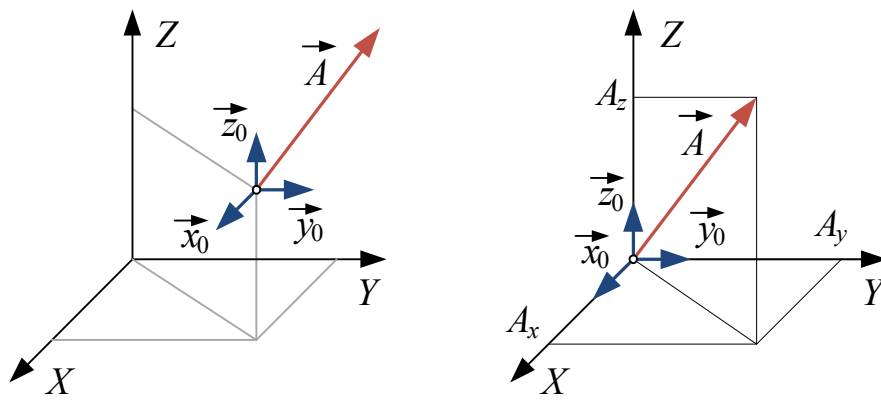
$$\vec{A} \pm \vec{B} = \vec{x}_0 (A_x \pm B_x) + \vec{y}_0 (A_y \pm B_y) + \vec{z}_0 (A_z \pm B_z). \quad (1.13)$$

The dot product of vectors \vec{A} and \vec{B} is defined:

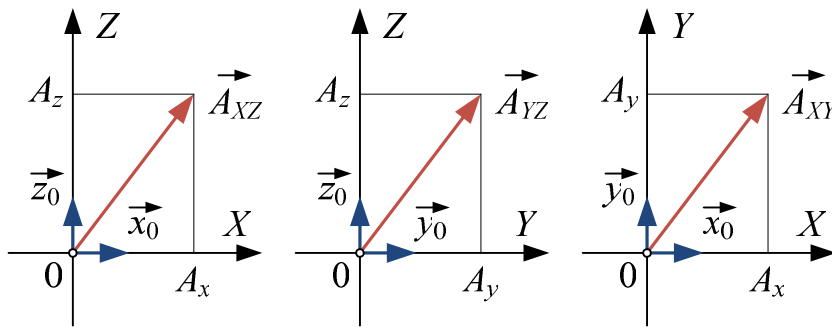
$$(\vec{A}, \vec{B}) \equiv \vec{A} \cdot \vec{B} = AB \cos \left(\vec{A}, \hat{\vec{B}} \right),$$

where $\left(\vec{A}, \hat{\vec{B}} \right)$ is a corner between vectors \vec{A}, \vec{B} ,

or
$$(\vec{A}, \vec{B}) = A_x B_x + A_y B_y + A_z B_z. \quad (1.14)$$

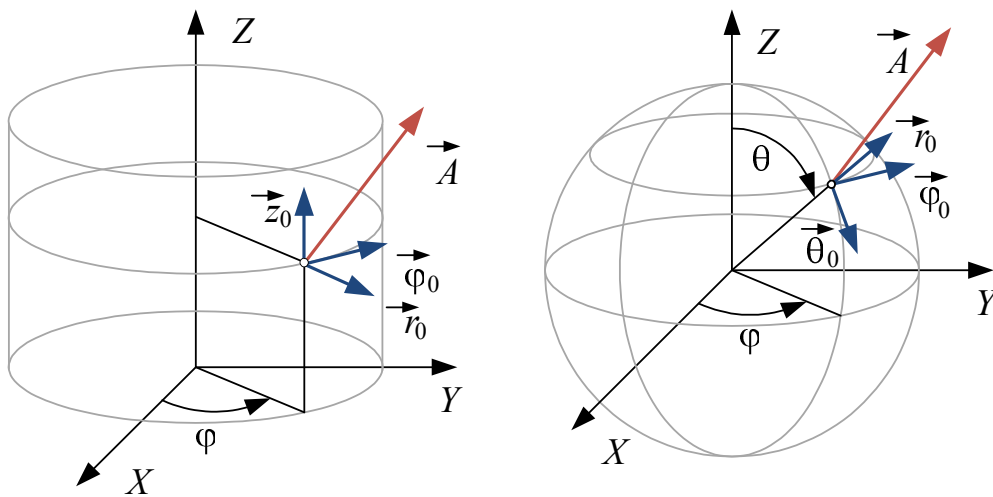


a)

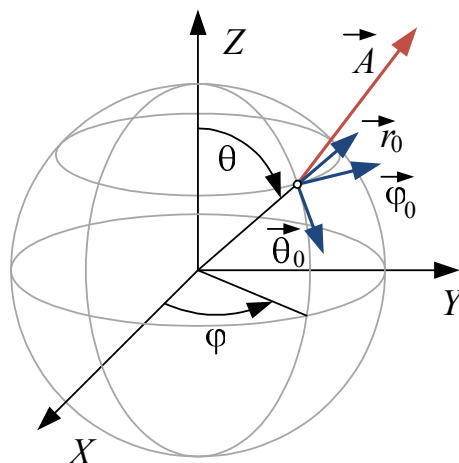


b)

Figure 1.2 – Vector \vec{A} representation: a) – in space; b) – projections of vector \vec{A} on the planes of the Cartesian coordinates



a)



b)

Figure 1.3 – Vector \vec{A} representation: a) – in the cylindrical coordinates; b) – in spherical coordinates

The cross product of vectors \vec{A} and \vec{B} is defined:

$$[\vec{A}, \vec{B}] \equiv \vec{A} \times \vec{B} = \vec{n}_0 AB \sin(\angle \vec{A}, \vec{B}),$$

where \vec{n}_0 is a unit vector (ort) normal to a plane containing vectors \vec{A} and \vec{B} and moreover vectors \vec{A}, \vec{B} and \vec{n}_0 are mutually perpendicular and form “right-hand triple”,

$$\text{or } [\vec{A}, \vec{B}] = \vec{x}_0(A_y B_z - A_z B_y) + \vec{y}_0(A_z B_x - A_x B_z) + \vec{z}_0(A_x B_y - A_y B_x). \quad (1.15)$$

Expressions for the sum (difference), scalar and cross product of vectors \vec{A} and \vec{B} , presented in cylindrical, spherical and other orthogonal coordinate systems, are similar to corresponding expressions (1.13), (1.14) and (1.15).

The vector analysis is used for research of scalar and vector fields featured by scalar Ψ and vector \vec{F} functions accordingly. The basic operations of the vector analysis: a scalar field gradient, a divergence and curl of vector field.

The **gradient** of scalar function Ψ is defined in corresponding coordinates by the following expressions:

$$\text{in Cartesian} \quad \text{grad} \Psi = \vec{x}_0 \frac{\partial \Psi}{\partial x} + \vec{y}_0 \frac{\partial \Psi}{\partial y} + \vec{z}_0 \frac{\partial \Psi}{\partial z}; \quad (1.16)$$

$$\text{in cylindrical} \quad \text{grad} \Psi = \vec{r}_0 \frac{\partial \Psi}{\partial r} + \vec{\phi}_0 \frac{1}{r} \frac{\partial \Psi}{\partial \phi} + \vec{z}_0 \frac{\partial \Psi}{\partial z}; \quad (1.17)$$

$$\text{in spherical} \quad \text{grad} \Psi = \vec{r}_0 \frac{\partial \Psi}{\partial r} + \vec{\theta}_0 \frac{1}{r} \frac{\partial \Psi}{\partial \theta} + \vec{\phi}_0 \frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial \phi}. \quad (1.18)$$

The **divergence** of the vector function \vec{F} is defined in corresponding coordinates by the following expressions:

$$\text{in Cartesian} \quad \text{div} \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}; \quad (1.19)$$

$$\text{in cylindrical} \quad \text{div} \vec{F} = \frac{1}{r} \frac{\partial}{\partial r}(r F_r) + \frac{1}{r} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}; \quad (1.20)$$

in spherical $\operatorname{div} \vec{F} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (F_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi}$. (1.21)

The *curl (vortex)* of the vector function \vec{F} is defined in corresponding coordinates by the following expressions:

in Cartesian $\operatorname{rot} \vec{F} = \vec{x}_0 \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + \vec{y}_0 \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + \vec{z}_0 \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$; (1.22)

in cylindrical $\operatorname{rot} \vec{F} = \vec{r}_0 \left(\frac{1}{r} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) + \vec{\phi}_0 \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r} \right) + \vec{z}_0 \left(\frac{1}{r} \frac{\partial}{\partial r} (r F_\phi) - \frac{1}{r} \frac{\partial F_r}{\partial \phi} \right)$; (1.23)

in spherical $\operatorname{rot} \vec{F} = \vec{r}_0 \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta F_\phi) - \frac{\partial F_\theta}{\partial \phi} \right] + \vec{\theta}_0 \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r} (r F_\phi) \right] + \vec{\phi}_0 \frac{1}{r} \left[\frac{\partial}{\partial r} (r F_\theta) - \frac{\partial F_r}{\partial \theta} \right]$. (1.24)

1.1.3 Key questions

- 1) What is a complex number?
- 2) What numbers are called complex conjugate, how are the complex conjugate numbers designated?
- 3) What is a complex plane?
- 4) What is algebraic and exponential forms of complex numbers, real and imaginary part of complex numbers, its module and argument and how do they relate to each other?
- 5) Explain the expressions of transformation of a complex number from the algebraic form into exponential one and back.
- 6) How the addition and division of complex numbers is done.
- 7) How the subtraction and multiplication of complex numbers is done.
- 8) Define a vector quantity.
- 9) What is general and coordinate representation of vectors (in Cartesian, cylindrical and spherical coordinates).
- 10) Explain rules of addition (subtraction) of vectors.
- 11) What is dot product of vectors?
- 12) What is cross product of vectors?
- 13) What is a gradient of scalar function? Write down the expression for

evaluation of gradient of scalar function in Cartesian coordinates.

14) What is a divergence of vector function? Write down the expression for evaluation of divergence of vector function in Cartesian coordinates.

15) What is a curl of vector function? Write down the expression for evaluation of a curl of the vector function in Cartesian coordinates.

1.1.4 Problems and methodical recommendations to their solutions

Problem 1. Two complex numbers are given $\dot{a} = 2e^{i\frac{\pi}{4}}$ and $\dot{b} = 3e^{-i\frac{\pi}{3}}$. Transform the numbers into the algebraic form. Find the sum, difference, product and the ratio of the two numbers. Find modules, arguments, the real and imaginary parts of the numbers. Arrange complex numbers on a complex plane (both given complex numbers and derived during calculations).

Problem 2. Two complex numbers are given $\dot{a} = 2 - i3$ and $\dot{b} = 3 + i$. Transform the numbers into the exponential form. Find the sum, difference, product and the ratio of the two numbers. Find modules, arguments, the real and imaginary parts of the numbers. Arrange complex numbers on a complex plane (both given complex numbers and derived during calculations).

Methodical recommendations to the solution of problems 1 and 2. For transformation of the complex number presented in the exponential form into algebraic and vice versa one needs to use expressions (1.5) and (1.3), (1.4) accordingly. To find the sum, difference, product and the ratio one needs to use expressions (1.6), (1.7) and (1.8) accordingly. The module, argument, real and imaginary parts of a complex number are the factors or addends in corresponding expressions (1.2) and (1.1).

To show a complex number on a complex plane one may use an example, as is shown in fig. 1.1.

Problem 3. Two vectors are set: $\vec{A} = 3\vec{y}_0$ and $\vec{B} = \vec{y}_0 - 4\vec{z}_0$, where $\vec{x}_0, \vec{y}_0, \vec{z}_0$ are orthonormal bases in Cartesian coordinates. Find the sum and difference of vectors, their dot and cross products. Plot these vectors on plane YOZ.

Problem 4. Two vectors are set: $\vec{A} = 2\vec{x}_0 + 3\vec{y}_0 - \vec{z}_0$ and $\vec{B} = 3\vec{x}_0 - \vec{y}_0 - 2\vec{z}_0$, where $\vec{x}_0, \vec{y}_0, \vec{z}_0$ are orthonormal bases in Cartesian coordinates. Find the sum and difference of the vectors, their dot and cross products. Plot vectors $\vec{A} + \vec{z}_0$ and $\vec{B} + 2\vec{z}_0$ on plane XOY.

Methodical recommendations to the solution of problems 3 and 4. For finding of sum or difference of vectors one may use the expression (1.13), dot and cross products of vectors — expressions (1.14) and (1.15) accordingly.

For the plotting of vectors on a corresponding plane one needs using an example, as is shown in fig. 1.2,b.

Problem 5. Scalar function is set $f(x, y, z) = x^2 + 2y + 3z$. Calculate gradient of this function.

Problem 6. The vector function is set $\vec{F}(x, y, z) = 2x\vec{x}_0 - 2y\vec{y}_0 + 3z^2\vec{z}_0$, where $\vec{x}_0, \vec{y}_0, \vec{z}_0$ are ors in Cartesian coordinates. Calculate divergence of this function and find, where does the function have source or sink.

Problem 7. The vector function is set $\vec{F}(x, y, z) = 2y\vec{x}_0 - 2x\vec{y}_0 + \vec{z}_0$, where $\vec{x}_0, \vec{y}_0, \vec{z}_0$ are ors in Cartesian coordinates. Calculate curl of this function.

Methodical recommendations to the solution of problems 5,6 and 7. For evaluation of a gradient of scalar function $f(x, y, z)$ in Cartesian coordinates one needs using expression (1.16) in which instead of Ψ one needs to substitute $f(x, y, z)$ and find derivatives on corresponding coordinates.

For evaluation of divergence and curl of the vector function $\vec{F}(x, y, z)$ in Cartesian coordinates one needs to use expressions (1.19) and (1.22) accordingly in which one needs to find derivatives on corresponding coordinates and then find the sum and difference of the terms accordingly.

Streamlines of the vector field featured by the vector function $\vec{F}(x, y, z)$ begin (come to an end) in those points of space, where $\text{div}\vec{F} \neq 0$. Points in which streamlines begin ($\text{div}\vec{F} > 0$), are called **sources**, and points in which streamlines come to an end ($\text{div}\vec{F} < 0$), are called **sinks**.

1.2 Basic postulates of electromagnetism

1.2.1 Goal of the practical training

Recovery of basics of electromagnetism; getting skills in solving of problems.

1.2.2 Key points

A particular case of electromagnetic field is electrostatic field resulting from a system of fixed charges. Thus **electrostatic field means the field of fixed electrical charges**.

For a quantitative estimation of the electric field in general and of electrostatic field in particular a physical quantity of electric field intensity \vec{E} is introduced from the Coulomb's law.

Coulomb's law is fundamental law of electrical interaction. The force \vec{F}_{12} between two charges q_1 and q_2 is directed along the line joining the charges and is directly proportional to the value of the charges and inversely proportional to the square of the distance between them (fig. 1.4).

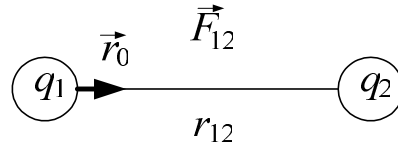


Figure 1.4 – Graphical interpretation of Coulomb's law

$$\vec{F}_{12} = \vec{r}_0 \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2}, \quad (1.25)$$

where \vec{F}_{12} is force which acts on charge q_1 from the side of charge q_2 in vacuum;

\vec{r}_0 is vector directed from q_1 to q_2 ;

r_{12} is distance between q_1 and q_2 ;

ϵ_0 is permittivity of free space

$$\epsilon_0 = 10^{-9}/36\pi = 8,854 \cdot 10^{-12}, \text{ F/m.}$$

The electric field intensity is the ratio of the force \vec{F}_{12} acting on the charge q_1 to the value of the point test charge q_2 . The electric field is independent on the test charge q_2 and characterizes electric field in vacuum for the field created by the charge q_1 in the point where the test charge q_2 is located.

$$\vec{E} = \frac{\vec{F}_{12}}{q_2} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}^2} \vec{r}_0, \quad \frac{\text{V}}{\text{m}}. \quad (1.26)$$

The direction of the electric field intensity vector \vec{E} determines the direction of the force acting on a positive charge placed at a particular point in the field.

For example if the field is created due to the positive charge q_1 the intensity vector \vec{E} is directed along the radius vector \vec{r}_0 from the charge to the outer space, if the field is created due to the negative charge q_1 , the intensity vector \vec{E} is directed to the charge.

Thus electric field intensity is a vector quantity equal to the force which a unit positive point charge placed at a given point of the field experiences from the side of electric field.

The mathematical relationship between the flux of electric field through a closed surface and charge in the volume bounded by this surface is established on the basis of the Gauss theorem. The physical basis of the Gauss theorem is Coulomb's law or otherwise Gauss theorem is an integral formulation of Coulomb's law.

Gauss theorem for electric field intensity in vacuum states: the flow of the electric field intensity vector \vec{E} through any randomly selected closed surface S is proportional to the total charge Q in a volume V bounded by this surface S

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} Q = \frac{1}{\epsilon_0} \int_V \rho \, dV, \quad (1.27)$$

where $\oint_S \vec{E} \cdot d\vec{S}$ is the flow of the electric field intensity vector \vec{E} through closed surface S ;

$Q = \int_V \rho \, dV$, Q is total charge within the volume V ;

ρ , C/m^3 is volume density of charge Q .

Equation (1.27) is the electrostatic Gauss theorem in integral form for a continuous charge, according to which the flow of the electric field intensity vector \vec{E} through a closed surface is independent on the charge distribution (disposition of charges) within the volume V and can be considered as a measure of the amount of electric charge Q in a volume limited by surface S .

Electrostatic Gauss theorem in differential form is as follows:

$$\text{div } \vec{E} \equiv \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad (1.28)$$

where ∇ is del operator or nabla-vector or Hamiltonian which is symbolic differential vector operator and in rectangular coordinates has the following form:

$$\nabla = \vec{x}_0 \frac{\partial}{\partial x} + \vec{y}_0 \frac{\partial}{\partial y} + \vec{z}_0 \frac{\partial}{\partial z}.$$

The divergence of vector \vec{E} characterizes the features of the electrostatic field in the places where either $\rho \neq 0$ or $\rho = 0$. Positive charge can be considered as a source of electrostatic field streamlines so the streamlines begin at positive charges. The negative charge is a sink of electrostatic field streamlines where they terminate (fig. 1.5).

If in a certain volume V (fig. 1.6) volume charge density is not equal to zero ($\rho \neq 0$) then the surface bounding this volume V , is threaded by streamlines of vector \vec{E} which are divergent in space ($\rho > 0$) or convergent ($\rho < 0$).

In the region of space where there are no charges ($\rho = 0$), the lines of

vector \vec{E} neither start nor end (fig. 1.7), any element of such a space the field lines go through only but neither diverge from it nor converge to it. The electrostatic field in the space where $\rho = 0$ and respectively $\text{div} \vec{E} = 0$ is called solenoidal.

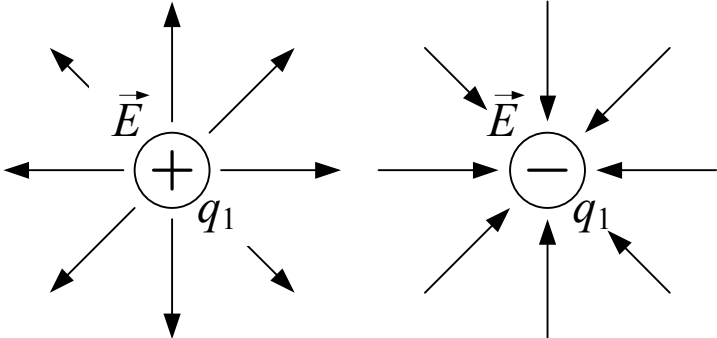


Figure 1.5 – Source and sink of electrostatic field

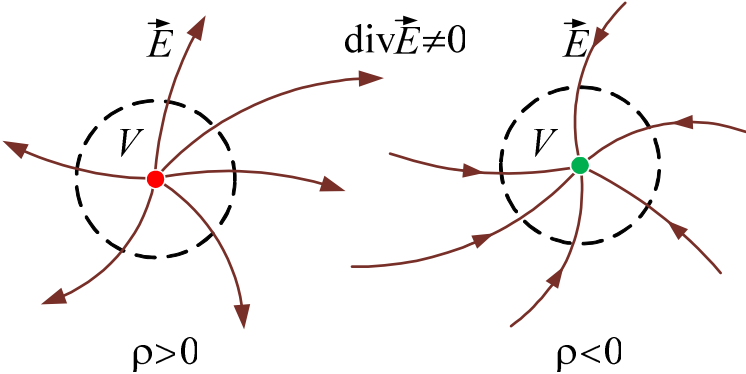


Figure 1.6 – Graphical interpretation of electrostatic Gauss theorem for $\text{div} \vec{E} \neq 0$

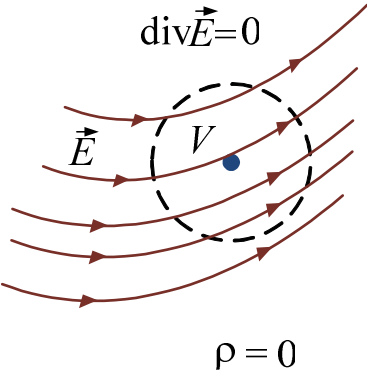


Figure 1.7 – Graphical interpretation of electrostatic Gauss theorem for $\text{div} \vec{E} = 0$

When replacing the vacuum by dielectric medium under the influence of electric field the medium gets polarized (positive charges in the dielectric displace in the direction of the electric field lines and the associated negative charges move against the direction of field). The influence of the dielectric on the electric field can be described by introducing an additional charge, which leads to a change in the flow of the electric field intensity vector \vec{E} . With that change a new vector \vec{D} is introduced. \vec{D} is electric displacement vector. For a linear homogeneous medium electric displacement vector is associated with the electric field intensity vector by expression

$$\vec{D} = \epsilon_a \vec{E}, \quad (1.29)$$

where $\epsilon_a = \epsilon \epsilon_0$ is absolute permittivity of the medium;
 ϵ is relative permittivity of the medium;
 ϵ_0 is permittivity of vacuum

$$\epsilon_0 = 10^{-9}/36\pi = 8,854 \cdot 10^{-12}, \text{ F/m.}$$

Circulation of the electrostatic field \vec{E} along any closed loop L is zero

$$\oint_L \vec{E} \, d\vec{l} = 0.$$

The field with this property is called potential. The vanishing of the circulation of the vector field means that lines of electrostatic fields can not be closed, they start and end on charges (respectively positive or negative) or go to infinity.

Magnetostatic field as well as the electrostatic one is a special case (or approximation) of electromagnetic field. The sources of magnetic field are moving electric charges (currents). The magnetic field arises in the space surrounding the conductor. The magnetic field of the permanent magnet is also generated by electrical micro-currents circulating within the molecules of matter (Ampere's hypothesis). *Magnetostatic field is field produced by constant electric currents and permanent magnets (static magnetic dipoles).*

To quantify magnetic field at a given point in space, the physical quantity of magnetic flux density \vec{B} is introduced which is a vector quantity that determines the force \vec{F} (Lorentz force) exerted by the magnetic field on the charge q moving with the speed \vec{v} (fig. 1.8).

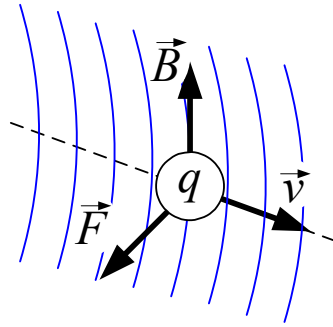


Figure 1.8 – Graphical interpretation of Lorentz force

In this case, the Lorentz force \vec{F} is defined from the expression

$$\vec{F} = q[\vec{v}, \vec{B}];$$

$$|\vec{F}| = F = q|\vec{v}||\vec{B}|\sin \alpha, \quad F_{\max} = q|\vec{v}||\vec{B}|, \quad \text{for } \alpha = \pi/2,$$

where $[\vec{v}, \vec{B}]$ is vector product of vectors \vec{v} and \vec{B} ;

α is angle between the velocity vector \vec{v} and the magnetic flux density vector \vec{B} .

The value of magnetic flux density $|\vec{B}|$ equals to the ratio of the maximum force F_{\max} acting on the charge q due to the magnetic field at a given point to the product of the charge q and its speed $|\vec{v}|$

$$|\vec{B}| = \frac{F_{\max}}{q|\vec{v}|}, \quad \text{T},$$

or, the value of the magnetic flux density $|\vec{B}|$ is numerically equal to the force with which the magnetic field acts on unit positive point charge moving with unit velocity perpendicularly to the lines of force.

The direction of magnetic flux density vector \vec{B} is perpendicular to the direction of speed \vec{v} and force \vec{F} , moreover the vectors \vec{v}, \vec{B} and \vec{F} form a right-handed system.

Magnetic field intensity vector \vec{H} is introduced axiomatically resulting from the influence of magnetic field on the atoms and molecules of matter and vacuum. In the matter magnetic flux density vector \vec{B} and magnetic field intensity vector \vec{H} are related by expression

$$\vec{B} = \mu_a \vec{H}, \quad (1.30)$$

where $\mu_a = \mu\mu_0$ is absolute permeability of the media;
 μ is relative permeability of the media;
 μ_0 is permeability of vacuum

$$\mu_0 = 4\pi \cdot 10^{-7} = 1,257 \cdot 10^{-6} \text{ H/m.}$$

In the magnetic field the stream-lines of the magnetic flux density \vec{B} are continuous, they have neither start nor end that is they are always closed upon themselves and never dropped. Therefore magnetic flux through any closed surface is equal to zero (the principle of the continuity of the magnetic flux)

$$\oint_S \vec{B} \cdot d\vec{S} = 0. \quad (1.31)$$

Fields that have this property are called eddy. The reason for this phenomenon is absence of magnetic charges (monopoles) in nature that could be the sources of magnetic field.

1.2.3 Key questions

- 1) What is electrostatic field?
- 2) What is the electric field intensity vector \vec{E} ?
- 3) What is the source of stream-lines of electrostatic field \vec{E} ?
- 4) What is the sink of stream-lines of electrostatic field \vec{E} ?
- 5) What is determined by electric displacement vector \vec{D} and how is it related to the vector of the electric field intensity \vec{E} ?
- 6) Which fields are called potential?
- 7) What is magnetostatic field?
- 8) What is the magnetic flux density vector \vec{B} ?
- 9) What is determined by magnetic field intensity vector \vec{H} and how is it related to the vector of magnetic flux density \vec{B} ?
- 10) Which fields are called eddy?

1.2.4 Problems and methodical recommendations to their solutions

Problem. Point positive charge $q = 2,9 \cdot 10^{-10}$ C is placed in the center of Cartesian coordinate system. Find the electric field intensity \vec{E} at a point $A(x, y, z)$, created by the charge q and specify the direction of the vector \vec{E} .

Cartesian coordinates of point A are the following: $x = 0,2$ m, $y = 0,3$ m, $z = 0,4$ m, the space under consideration is vacuum.

Methodical recommendations to problem solution. To determine the electric field intensity vector \vec{E} one can use expression (1.26), in which instead of q_1 one needs to substitute the given value of a point charge q , and in order to determine the distance between the point charge and the point under consideration one may use the expression

$$r_{12} = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2},$$

where x, y, z are Cartesian coordinates of point A ;

x_0, y_0, z_0 are Cartesian coordinates of point charge (as a point charge, according to the statement of the problem, is located in the center of the Cartesian coordinate system, $x_0 = 0, y_0 = 0, z_0 = 0$).

The direction of the electric field intensity vector can be determined according to the sign of the charge (positive or negative).

2 BASICS OF TRANSMISSION LINES THEORY

2.1 Parameters and performances of a transmission line with finite length

2.1.1 Goal of the practical training

Study of the transmission lines basics; acquisition of skills of the problems solution.

2.1.2 Key points

Any transmission line (two-wire line, coaxial line, waveguide or other) may be represented in a form of equivalent line with distributed parameters. **Line with distributed parameters** is a transmission line formed by two parallel conductors the length of which exceeds the wavelength of electromagnetic oscillations transmitted by the line and the distance between the conductors is considerably smaller than the wavelength. There are three operation modes of lines with distributed parameters: **traveling waves** when the transferred energy is fully absorbed by the load; **standing waves** when the transferred energy is fully reflected back from the end of the line and intermediate mode — **mixed waves**.

When a complex load $\dot{Z}_L = R_L + iX_L$ is connected to a transmission line the line works in the mixed waves operating mode. According to the wave model an electromagnetic wave in any section of a transmission line is possible to be presented in the form of superposition of incident $\dot{E}_m^{\text{inc}}(z)$, and $\dot{H}_m^{\text{inc}}(z)$ and reflected $\dot{E}_m^{\text{ref}}(z)$, $\dot{H}_m^{\text{ref}}(z)$ waves of electric and magnetic field intensities, accordingly

$$\begin{cases} \dot{E}_\Sigma(z) = \dot{E}_m^{\text{inc}}(z) + \dot{E}_m^{\text{ref}}(z); \\ \dot{H}_\Sigma(z) = \dot{H}_m^{\text{inc}}(z) + \dot{H}_m^{\text{ref}}(z). \end{cases} \quad (2.1)$$

Let's transform (2.1) to a view

$$\begin{cases} \dot{E}_\Sigma(z) = \dot{E}_m^{\text{inc}}(z) \left[1 + \dot{E}_m^{\text{ref}}(z) / \dot{E}_m^{\text{inc}}(z) \right]; \\ \dot{H}_\Sigma(z) = \dot{H}_m^{\text{inc}}(z) \left[1 + \dot{H}_m^{\text{ref}}(z) / \dot{H}_m^{\text{inc}}(z) \right]. \end{cases} \quad (2.2)$$

Let's introduce **complex reflection coefficient** on electric field $\dot{\Gamma}(z) = \dot{E}_m^{\text{ref}}(z) / \dot{E}_m^{\text{inc}}(z)$ then taking into account orthogonality of electrical and magnetic field vectors in a transmission line it is possible to write down and

$\dot{H}_m^{\text{ref}}(z)/\dot{H}_m^{\text{inc}}(z) = -\dot{\Gamma}(z)$ and rewrite (2.2):

$$\begin{cases} \dot{E}_\Sigma(z) = \dot{E}_m^{\text{inc}}(z)[1 + \dot{\Gamma}(z)] ; \\ \dot{H}_\Sigma(z) = \dot{H}_m^{\text{inc}}(z)[1 - \dot{\Gamma}(z)]. \end{cases} \quad (2.3)$$

The complex reflection coefficient for electric field is defined from the expression:

$$\dot{\Gamma}(z) = \dot{\Gamma}_L e^{-i2\beta z}, \quad \dot{\Gamma}_L = \Gamma_L e^{i\varphi_L}, \quad (2.4)$$

where z is coordinate measured from a load \dot{Z}_L , located in section $z = 0$;

$\dot{\Gamma}_L$ is complex reflection coefficient in a place of insert of load \dot{Z}_L ;

Γ_L, φ_L are accordingly module and argument (phase) of reflection coefficient from a load \dot{Z}_L ;

$\beta = 2\pi/\lambda_w$ is longitudinal wave number or phase coefficient;

λ_w is a wave length of the microwave oscillations in a transmission line.

Thus, for the homogeneous lossless transmission line the equations (2.3) can be written down:

$$\begin{cases} \dot{E}_\Sigma(z) = |E_m^{\text{inc}}| \exp(i\beta z) [1 + \Gamma_L \exp[i(\varphi_L - 2\beta z)]]; \\ \dot{H}_\Sigma(z) = |H_m^{\text{inc}}| \exp(i\beta z) [1 - \Gamma_L \exp[i(\varphi_L - 2\beta z)]], \end{cases} \quad (2.5)$$

where amplitudes of incident waves $|E_m^{\text{inc}}|, |H_m^{\text{inc}}|$ don't depend on z .

The longitudinal distribution (distribution along the transmission line) of amplitudes of electrical $|\dot{E}_\Sigma(z)|$ and magnetic $|\dot{H}_\Sigma(z)|$ field intensities represents the greatest interest for study. The distribution can be calculated on the basis of (2.5) in a form:

$$\begin{cases} |\dot{E}_\Sigma(z)| = E_m^{\text{inc}} \sqrt{1 + \Gamma_L^2 + 2\Gamma_L \cos(\varphi_L - 2\beta z)} ; \\ |\dot{H}_\Sigma(z)| = H_m^{\text{inc}} \sqrt{1 + \Gamma_L^2 - 2\Gamma_L \cos(\varphi_L - 2\beta z)}. \end{cases} \quad (2.6)$$

The graphic representation of longitudinal distribution of the relative amplitudes of electrical $|\dot{E}_\Sigma(z)|/|E_m^{\text{inc}}|$ and magnetic $|\dot{H}_\Sigma(z)|/|H_m^{\text{inc}}|$ field intensities is shown in fig. 2.1.

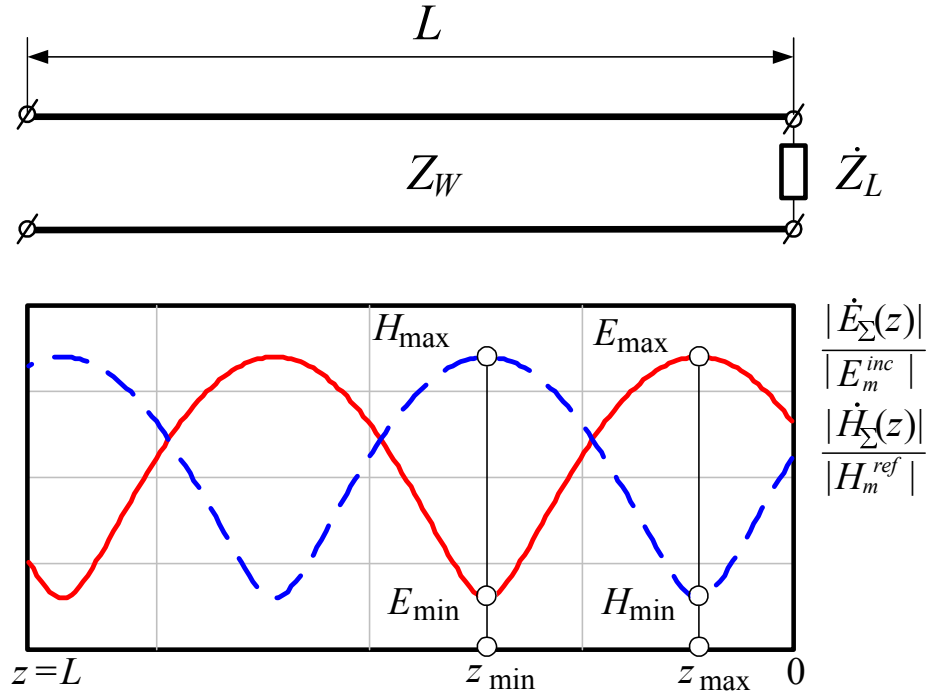


Figure 2.1 – Longitudinal distribution of the relative amplitudes of electrical and magnetic field intensities

Maximums of amplitudes of electric field intensity $E_{\max} = \left(\left| \dot{E}_{\Sigma}(z) \right| / \left| E_m^{\text{inc}} \right| \right)_{\max}$ are observed in those sections of a line z_{\max} termed also **as loop**, (fig. 2.1) in which the reflected wave is summed up with the incident wave in phase:

$$\varphi_L - 2\beta z_{\max} = 2\pi n, \text{ where } n = 0, 1, 2, \dots \quad (2.7)$$

Minimums of amplitudes of electric field intensity $E_{\min} = \left(\left| \dot{E}_{\Sigma}(z) \right| / \left| E_m^{\text{inc}} \right| \right)_{\min}$ correspond to anti-phase addition of waves in the sections z_{\min} called also **as nodes**, (fig. 2.1):

$$\varphi_L - 2\beta z_{\min} = \pm\pi + 2\pi n, \text{ where } n = 0, 1, 2, \dots \quad (2.8)$$

The neighboring minimums and maximums of amplitude distribution of electric intensity are located $\lambda_w/4$ apart from each other, therefore it is enough to find, for example, a situation of the first maximum of amplitude of an electric field intensity from (2.7)

$$\varphi_L = 2\beta z_{\max} = 4\pi z_{\max}/\lambda_w, \quad (2.9)$$

or

$$z_{\max}/\lambda_w = \varphi_L/4\pi. \quad (2.10)$$

Then

$$z_{\min} = z_{\max} \pm \lambda_w/4. \quad (2.11)$$

The peak values of an electric field intensity corresponding z_{\max} or z_{\min} (fig. 2.1), according to (2.7) are repeated every $\lambda_w/2$ and are equal:

$$\begin{cases} E_{\max} = 1 + \Gamma_L; \\ E_{\min} = 1 - \Gamma_L. \end{cases} \quad (2.12)$$

As one can see from fig. 2.1, distribution of relative amplitude of magnetic field intensity is completely the same in shape as distribution of relative amplitude of electric field intensity, but it is shifted on $\lambda_w/4$. Thus, the place of maximums z_{\max} of electric intensity amplitude corresponds to the place of minimums of magnetic intensity amplitude and vice versa.

Using traveling-wave ratio definition (TWR) or standing wave ratio definition (SWR), we derive:

$$K_{TW} = \frac{E_{\max}}{E_{\min}} = \frac{1 - \Gamma_L}{1 + \Gamma_L} \quad \text{or} \quad K_{SW} = \frac{E_{\max}}{E_{\min}} = \frac{1 + \Gamma_L}{1 - \Gamma_L}. \quad (2.13)$$

Then

$$\Gamma_L = \frac{1 - K_{TW}}{1 + K_{TW}} = \frac{K_{SW} - 1}{K_{SW} + 1}. \quad (2.14)$$

Due to electric intensity $\dot{E}_\Sigma(z)$ is proportional to a voltage in the given section z of a transmission line, and the magnetic intensity $\dot{H}_\Sigma(z)$ is proportional to a current flowing through the given section z of a transmission line, it is possible to write down expression for definition of the full impedance at a given section z of a transmission line on the basis of (2.3), (2.5)

$$\dot{Z}(z) = \frac{\dot{E}_\Sigma(z)}{\dot{H}_\Sigma(z)} = Z_w \frac{1 + \dot{\Gamma}(z)}{1 - \dot{\Gamma}(z)} = Z_w \frac{1 + \Gamma_L \exp[i(\varphi_L - 2\beta z)]}{1 - \Gamma_L \exp[i(\varphi_L - 2\beta z)]}, \quad (2.15)$$

where $Z_w = E_m^{\text{inc}}/H_m^{\text{inc}}$ is transmission line wave impedance.

From (2.15) follows that at $z = 0$ (section of load \dot{Z}_L)

$$\dot{Z}(z=0) = Z_w \frac{1 + \dot{\Gamma}(z=0)}{1 - \dot{\Gamma}(z=0)} = \dot{Z}_L,$$

$$\dot{Z}_L = Z_w \frac{1 + \dot{\Gamma}_L}{1 - \dot{\Gamma}_L} = Z_w \frac{1 + \Gamma_L \exp(i\varphi_L)}{1 - \Gamma_L \exp(i\varphi_L)}, \quad (2.16)$$

or

$$\dot{\Gamma}_L = \Gamma_L \exp(i\varphi_L) = \frac{\dot{Z}_L - Z_w}{\dot{Z}_L + Z_w}, \quad (2.17)$$

On the basis of (2.17) module Γ_L and argument φ_L of reflection coefficient in a place of a load \dot{Z}_L insert are defined from expressions:

$$\Gamma_L = \frac{|\dot{Z}_L - Z_w|}{|\dot{Z}_L + Z_w|} = \frac{\sqrt{(R_L - Z_w)^2 + X_L^2}}{\sqrt{(R_L + Z_w)^2 + X_L^2}}; \quad (2.18)$$

$$\varphi_L = \begin{cases} \tan^{-1}\left(\frac{X_L}{R_L - Z_w}\right) - \tan^{-1}\left(\frac{X_L}{R_L + Z_w}\right), & \text{when } R_L \geq Z_w; \\ \pi + \tan^{-1}\left(\frac{X_L}{R_L - Z_w}\right) - \tan^{-1}\left(\frac{X_L}{R_L + Z_w}\right), & \text{when } R_L < Z_w. \end{cases} \quad (2.19)$$

For a given length L of a piece of a transmission line the quantity $\dot{Z}(z=L)$ is called **full input impedance** \dot{Z}_{IN} of transmission line and is defined according to (2.15).

After transformation of (2.15) with the account of $z = L$, one can derive

$$\begin{aligned} \dot{Z}(z=L) = \dot{Z}_{IN} &= Z_w \frac{1 + \Gamma_L \exp[i(\varphi_L - 2\beta L)]}{1 - \Gamma_L \exp[i(\varphi_L - 2\beta L)]} = \\ &= Z_w \left[\frac{1 - \Gamma_L^2}{1 + \Gamma_L^2 - 2\Gamma_L \cos(\varphi_L - 2\beta L)} + i \frac{2\Gamma_L \sin(\varphi_L - 2\beta L)}{1 + \Gamma_L^2 - 2\Gamma_L \cos(\varphi_L - 2\beta L)} \right]. \end{aligned} \quad (1.20)$$

In fig. 2.2 the distribution of the full impedance along the transmission line (active part is shown by a continuous line and reactive part is shown by dashed

line) is shown, the values R_L, X_L and R_{IN}, X_{IN} are also shown. According to fig. 2.2 extreme values (maximums and minimums) of the active component $R(z)$ correspond to zero values of reactive component $X(z)$ of full impedance.

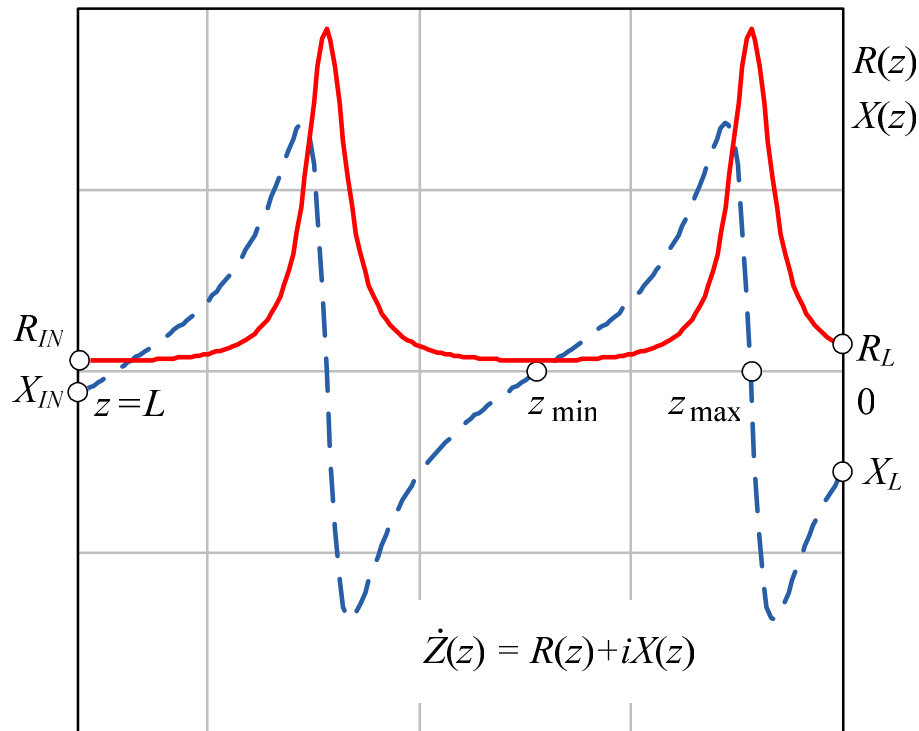


Figure 2.2 – Distribution of the full impedance (active and reactive components) along a transmission line

The full impedance in the $\dot{Z}(z) = R(z) + iX(z)$ at a given section z , impedance of a load $\dot{Z}_L = R_L + iX_L$ and full input transmission line impedance $\dot{Z}_{IN} = R_{IN} + iX_{IN}$ are generally complex numbers. A complex plane may be used for a graphical representation of complex numbers (fig. 1.1) however in practice the Smith's chart is often used (polar impedance chart), that is named after the Soviet scientist A. R. Volpert and the American engineer Ph. Smith who offered the chart independently from each other in 1939.

The Smith's chart is a circle in which two families of the orthogonal circles corresponding to geometrical places of points of the normalized resistances $R/Z_w = \text{const}$ and $X/Z_w = \text{const}$ are located. The use of normalized impedance allows to apply the chart for measurement of the full impedance connected in transmission lines with any wave impedance. On vertical diameter the values of R/Z_w corresponding to circles of equal active resistances going through them are placed. In places where equal reactance circles cross with an exterior circle the values X/Z_w are placed: in the left half of the chart there are negative values, in right half there are positive ones (fig. 2.3).

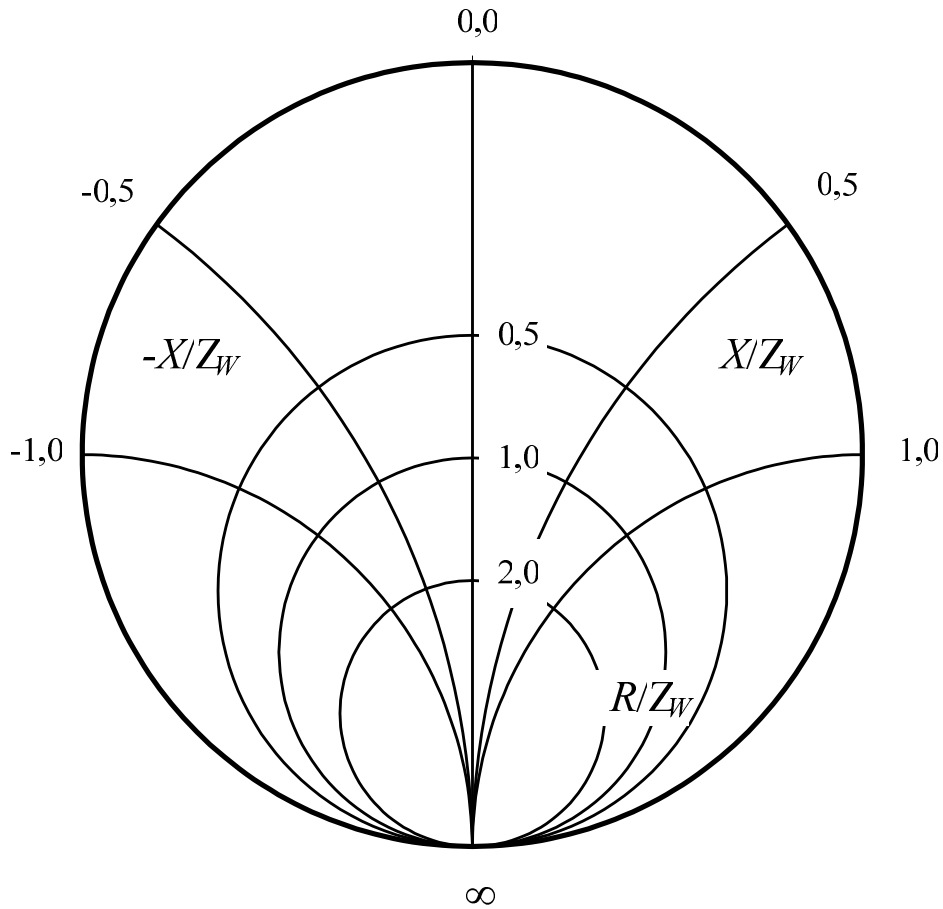


Figure 2.3 – The Smith's chart
(polar impedance chart)

The full impedance is defined on the Smith's chart in the following way. Find an intersection point of two orthogonal circles R/Z_W and X/Z_W . Values R/Z_W and X/Z_W , corresponding to the cross point, after multiplication by Z_W allow to determine $\dot{Z} = R + iX$.

As an example in fig. 2.4 two Smith's charts are shown with the values of the full normalized impedances $\dot{Z}/Z_W = R/Z_W + iX/Z_W$ plotted on them. In particular, the intersection point of dashed lines in fig. 2.4, *a* correspond to value $\dot{Z}/Z_W = 0,5 - i0,7$ and in fig. 2.4, *b* accordingly $\dot{Z}/Z_W = 0,7 + i0,8$.

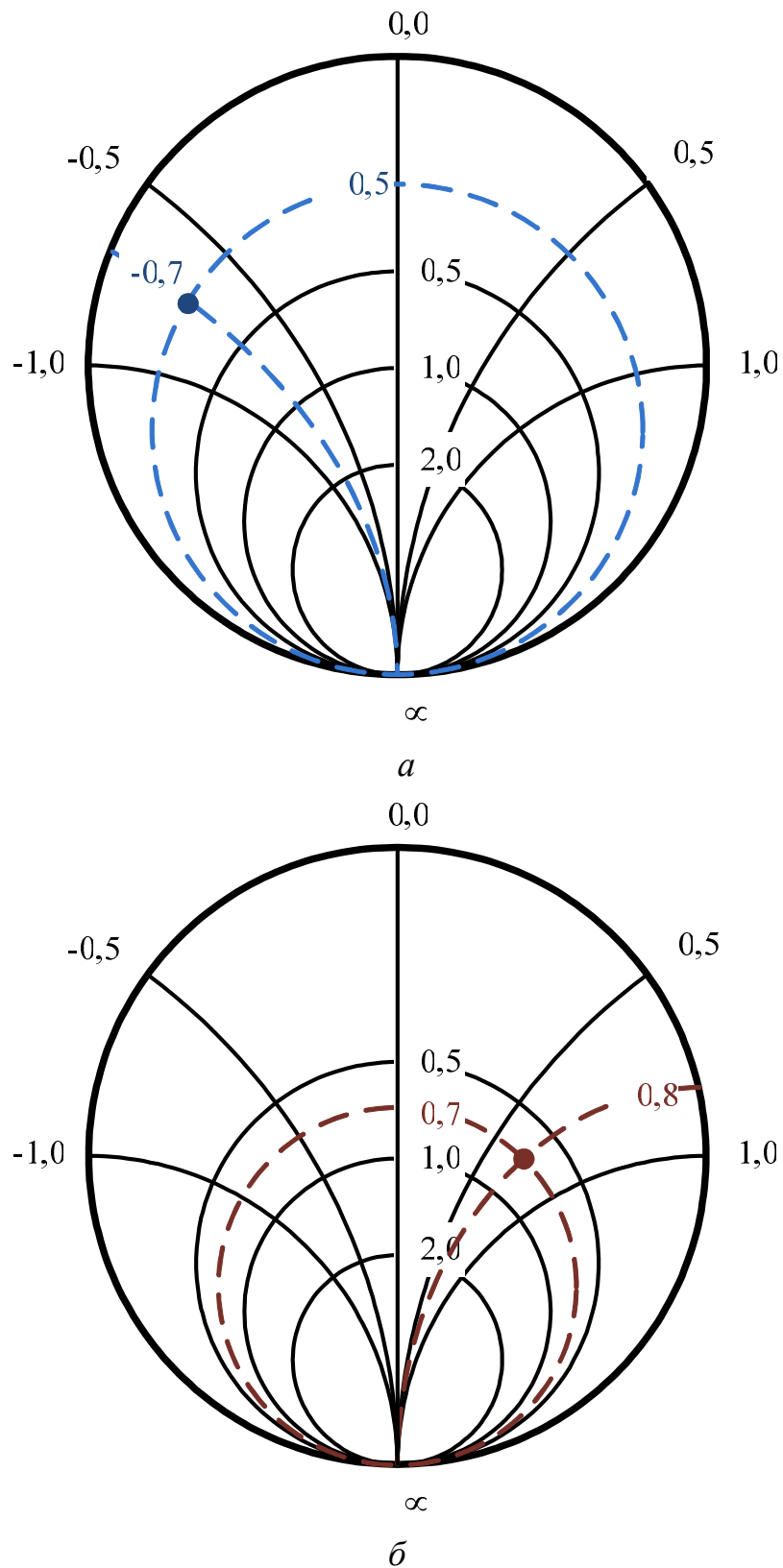


Figure 2.4 – Examples of numerical values of full impedances shown on Smith's chart

2.1.3 Key questions

- 1) What is complex reflection coefficient?
- 2) What is wave impedance of a transmission line?
- 3) What is traveling-wave ratio (TWR) and standing wave ratio (SWR)?
- 4) Explain concepts of “wave length”, “longitudinal wave number”.
- 5) Explain concepts of “node” and “loop”. How do the neighboring node and loop are located in the amplitude distribution?
- 6) Write down and explain the expression linking a traveling-wave ratio (standing wave ratio) and the module of load reflection coefficient.
- 7) Write down and explain the expression linking a complex reflection coefficient and impedance of the load.
- 8) Write down and explain the expression for full impedance of a transmission line in the given section.
- 9) Write down and explain the expression for full input impedance of a transmission line.
- 10) What is the reactive component of wave impedance in the extremes (maximums and minimums) of active part of the full impedance?
- 11) What is Smith’s chart?
- 12) Explain how full impedance is determined from a value shown on the Smith’s chart?

2.1.4 Problems and methodical recommendations to their solutions

Problem 1. Find complex reflection coefficient $\dot{\Gamma}_L$ from load \dot{Z}_L if complex amplitudes of the incident $\dot{E}_m^{\text{inc}}(z=0)$ and reflected $\dot{E}_m^{\text{ref}}(z=0)$ waves of electric field intensity on a load are equal accordingly 10 V/m and $5e^{i\pi/4}$ V/m. Calculate TWR and SWR in the line.

Problem 2. Find TWR and SWR in transmission line with a wave impedance $Z_W = 300 \Omega$, terminated with a load with impedance $\dot{Z}_L = 200 + i50, \Omega$.

Methodical recommendations to the solution to problems 1 and 2. In order to find complex reflection coefficient $\dot{\Gamma}_L$ from a load depending on the given data one may use expressions (2.16), (2.17) and (2.8) or expression $\dot{\Gamma}_L = \dot{E}_m^{\text{ref}}(z=0) / \dot{E}_m^{\text{inc}}(z=0)$. To find TWR and SWR it is enough to find the module of reflection coefficient from load Γ_L and use expressions (2.13).

Problem 3. The generator is connected to a two-wire lossless transmission line with a wave impedance $Z_W = 600 \Omega$ at frequency 60 MHz. The generator excites an incident wave of electric intensity in a line with amplitude $E_m^{\text{inc}} = 4$ V/m. The line is terminated with a load with impedance

$$\dot{Z}_L = 500 + i30, \Omega.$$

It is required to:

- find complex reflection coefficient from the load $\dot{\Gamma}_L$;
- find TWR and SWR in the transmission line;
- find value of amplitude $|\dot{E}_\Sigma(z)|/|E_m^{\text{inc}}|$ in nodes and loops of electric field intensity distribution;
- find a position of first two nodes and loops of electric field intensity;
- calculate and plot a graph of electric field intensity amplitude $|\dot{E}_\Sigma(z)|/|E_m^{\text{inc}}|$ dependence in a transmission line on a distance from the load.

Methodical recommendations to the solution of problem 3. To find complex reflection coefficient $\dot{\Gamma}_L$ from a load, and also TWR and SWR in a transmission line one may use methodical recommendations to the solution of problems 1 and 2.

To find position of nodes and loops, and also values of amplitude $|\dot{E}_\Sigma(z)|/|E_m^{\text{inc}}|$ in nodes and loops of electric field intensity taking into account known values of the module and argument (phase) of a reflection coefficient $\dot{\Gamma}_L$ from a load one may use expressions (2.10), (2.11) and (2.12).

For calculation of dependence of amplitude $|\dot{E}_\Sigma(z)|/|E_m^{\text{inc}}|$ of electric field intensity in a transmission line it is possible to use the expression (2.6), an example of the distribution graph that is shown in fig. 2.1.

Calculation and building of distribution graph may be implemented by means of specialized software, for example, MathCAD, MATLAB or other.

Problem 4. Calculate the input impedance of a transmission line with a wave impedance $Z_W = 50 \Omega$, shown as a point in the Smith's chart. The point in the Smith's chart is set by the teacher.

Methodical recommendations to the solution of problem 4. For evaluation of the full impedance of a transmission line $\dot{Z} = R + iX$ it is firstly necessary to find the active R/Z_W and reactive X/Z_W components of the normalized impedance. The procedure of calculation of active R/Z_W and reactive X/Z_W components of the normalized impedance is illustrated by the examples shown in fig. 2.4. Further the determined values of the active R/Z_W and reactive X/Z_W components of the normalized impedance are multiplied by numerical value of a wave impedance of a transmission line Z_W . The found complex number $\dot{Z} = R + iX$ is the full impedance of a transmission line with a wave impedance Z_W .

2.2 Matching of transmission lines with complex load

2.2.1 Goal of the practical training

Study of methods of the narrow-band matching of a line with a load; acquisition of skills of problems solution.

2.2.2 Key points

The matching is a maintenance in a transmission line of a mode, close to traveling waves in the given frequency band. When the demanded frequency band is narrow (practically no more than 5 % from centre frequency), the matching is called narrow-band, and matching in a wide frequency band is called broadband and even wide-band.

Now there are **two methods of narrow-band matching** used widely in practice: a method of transformation of impedances on the basis of the quarter wave transformer and a method of neutralization of reactance on the basis of Tatarinov's shunt.

The **method of transformation of impedances** is realized by insertion of a load $\dot{Z}_L = R_L + iX_L$ into transmission line via the transformer which is implemented on a basis of a piece of a line with other wave impedance Z_W^{tr} (fig. 2.5).

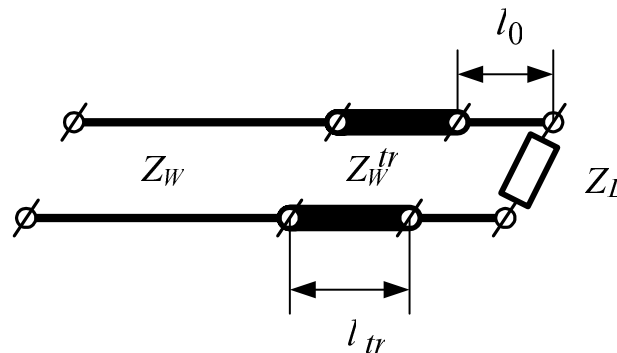


Figure 2.5 – Matching with quarter wave transformer

Matching is achieved when transformer's input impedance is equal to a transmission line wave impedance, that is

$$\operatorname{Re} Z(l_{tr}) = Z_W; \quad \operatorname{Im} Z(l_{tr}) = 0. \quad (2.21)$$

Using expressions (2.15) and (2.17), it is possible to derive a common solution of the equations (2.21) in a form:

$$\tan(\beta l_{tr}) = \sqrt{\frac{Z_W (Z_W - R_L)(R_L Z_W - R_L^2 - X_L^2)}{X_L Z_W}}; \quad (2.22)$$

$$Z_W^{tr} = \sqrt{\frac{Z_W (R_L Z_W - R_L^2 - X_L^2)}{Z_W - R_L}}. \quad (2.23)$$

For purely active load $\dot{Z}_L = R_L$ one gets $\tan(\beta l_{tr}) = \infty$, accordingly

$$l_{tr} = \lambda_w/4 + (\lambda_w/4)n, \text{ where } n = 0,1,2,\dots;$$

$$Z_W^{tr} = \sqrt{Z_W R_L}. \quad (2.24)$$

Usually the minimum length of a transformer $\lambda_w/4$ is chosen. Such transformer is called **quarter wave transformer**. It can be applied even for a complex load. Thus the transformer is inserted in such section of the basic transmission line in which the full impedance is purely active – in points of a minimum or maximum of distribution $R(z)$ (fig. 2.2).

In practice the nearest to a load extremum is chosen. Then the place of the transformer insertion is defined by $l_0 = z_{\max}$ or $l_0 = z_{\min}$ depending on which of the extremums is the nearest to a load, the transformer length is equal $l_{tr} = \lambda_w/4$ and its wave impedance Z_W^{tr} is defined on the basis of expressions:

when transformer is inserted in a voltage loop ($l_0 = z_{\max}$)

$$Z_W^{tr} = \sqrt{Z_W R_{\max}}, \text{ where } R_{\max} = Z_W K_{SW} = Z_W / K_{TW}, \quad (2.25)$$

when transformer is inserted in a voltage node ($l_0 = z_{\min}$)

$$Z_W^{tr} = \sqrt{Z_W R_{\min}}, \text{ where } R_{\min} = Z_W / K_{SW} = Z_W K_{TW}. \quad (2.26)$$

The **method of neutralization of the reactance**, realized on the basis of **Tatarinov's shunt** (fig. 2.6), is based on the following. The short-circuited piece of a transmission line has purely reactive input impedance which can be of any value and of any sign depending on the length of the shunt l_{sh} and its wave impedance Z_W^{sh} that allows to compensate any reactance of a load.

The short-circuited piece of a transmission line is connected in parallel into certain section $z = l_0$ (fig. 2.6) in which the active component of full admittance is equal to quantity, reciprocal to a transmission line wave impedance.

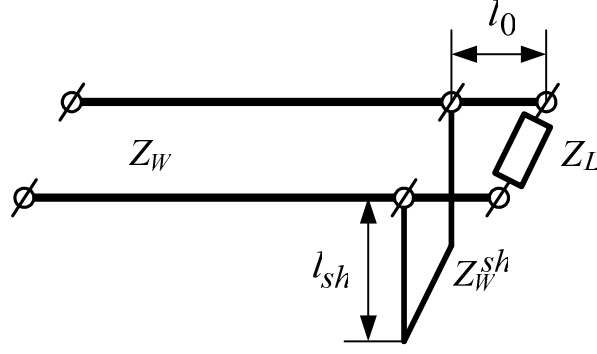


Figure 2.6 – Matching with a compensating reactance

Then the reactive component of full admittance can be compensated by a reactive admittance of the shunt (a short-circuited piece of a transmission line) taken with opposite sign:

$$\operatorname{Re}\left(\frac{1}{\dot{Z}_{tl}(l_0)}\right) = \frac{1}{Z_W}; \quad \operatorname{Im}\left(\frac{1}{\dot{Z}_{tl}(l_0)}\right) = -\operatorname{Im}\left(\frac{1}{\dot{Z}_{sh}(l_{sh})}\right), \quad (2.27)$$

where $Z_{tl}(l_0)^{-1}$ is admittance of a transmission line in section $z = l_0$ without shunt;

$$\dot{Z}_{tl}(l_0) = Z_W \frac{\dot{Z}_L + iZ_W \tan(\beta l_0)}{Z_W + i\dot{Z}_L \tan(\beta l_0)}; \quad (2.28)$$

$\dot{Z}_{sh}(l_{sh})$ is a shunt input impedance;

$$\dot{Z}_{sh}(l_{sh}) = iZ_W^{sh} \tan(\beta l_{sh}); \quad (2.29)$$

Let's solve equations system (2.27) taking into account (2.28) and (2.29), the following expressions are derived:

$$\tan(\beta l_0) = \frac{X_L}{R_L - Z_W} + \frac{R_L}{R_L - Z_W} \sqrt{\frac{(R_L - Z_W)^2 + X_L^2}{R_L Z_W}}; \quad (2.30)$$

or

$$l_0/\lambda_w = \frac{1}{2\pi} \tan^{-1}\left(\frac{X_L}{R_L - Z_W} + \frac{R_L}{R_L - Z_W} \sqrt{\frac{(R_L - Z_W)^2 + X_L^2}{R_L Z_W}}\right); \quad (2.31)$$

$$\tan(\beta l_{sh}) = \frac{Z_W}{Z_W^{sh}} \sqrt{\frac{R_L Z_W}{(R_L - Z_W)^2 + X_L^2}}. \quad (2.32)$$

or
$$l_{sh}/\lambda_w = \frac{1}{2\pi} \tan^{-1} \left(\frac{Z_W}{Z_W^{sh}} \sqrt{\frac{R_L Z_W}{(R_L - Z_W)^2 + X_L^2}} \right). \quad (2.33)$$

If as a result of calculation in (2.31) $l_0/\lambda_w < 0$ (case $R_L < Z_W$ and $X_L > 0$) the true value l_0^*/λ_w is determined from expression $l_0^*/\lambda_w = l_0/\lambda_w + 0.5$.

In case $X_L = 0$ expressions (2.30) – (2.33) become simpler:

$$\tan(\beta l_0) = \sqrt{R_L/Z_W}; \quad (2.34)$$

or
$$l_0/\lambda_w = \frac{1}{2\pi} \tan^{-1}(\sqrt{R_L/Z_W}); \quad (2.35)$$

$$\tan(\beta l_{sh}) = \frac{Z_W}{Z_W^{sh}} \sqrt{\frac{R_L Z_W}{(R_L - Z_W)^2}}. \quad (2.36)$$

or
$$l_{sh}/\lambda_w = \frac{1}{2\pi} \tan^{-1} \left(\frac{Z_W}{Z_W^{sh}} \sqrt{\frac{R_L Z_W}{(R_L - Z_W)^2}} \right). \quad (2.37)$$

Usually for convenience of engineering implementation and for better electrical strength the shunt is made on a basis of a piece of a similar transmission line with the same design parameters. Therefore a shunt wave impedance is equal to wave impedance of a transmission line $Z_W^{sh} = Z_W$.

2.2.3 Key questions

- 1) Explain what is «matching of transmission line with a load»?
- 2) Why there is a necessity of matching of the line with a load?
- 3) What is the difference between narrow-band and broadband matching of a load with a line?
- 4) Explain the essence of matching method of a line with a load by means of quarter wave transformer.
- 5) In which case the quarter wave transformer is necessary to be inserted directly next to load?
- 6) Is it possible to use the quarter wave transformer for the matching of complex impedances?
- 7) State the idea of matching by means of Tatarinov's shunt.
- 8) What is shunt? How does input impedance of a shunt change at change of its relative length?

2.2.4 Problems and methodical recommendations to their solutions

Problem 1. Transmission line with the wave impedance $Z_W = 100 \Omega$ is connected to the load $Z_L = 50 + i100 \Omega$. Find the power P_L dissipated in a load Z_L if root-mean-square value (rms value) of voltage on the input of transmission line is $U_{IN} = 60 \text{ V}$. Find the maximum power $P_{L_{\max}}$ which will be dissipated in a load Z_L in case of its full matching with a transmission line.

Methodical recommendations to the solution of a problem 1. The power P_L dissipated in a load Z_L , for known rms value of a voltage at transmission line input U_{IN} can be calculated from expression

$$P_L = \frac{U_{IN}^2}{Z_W} (1 - \Gamma_L^2),$$

where Γ_L is the module of reflection coefficient.

For known values of a wave impedance Z_W of transmission line and complex impedance of a load $Z_L = R_L + iX_L$ module of reflection coefficient can be found according to (2.18).

The maximum power $P_{L_{\max}}$ which is dissipated in a load Z_L in case it is fully matched with a transmission line ($\Gamma_L = 0$) and known rms value of a voltage at transmission line input U_{IN} can be calculated from expression

$$P_{L_{\max}} = \frac{U_{IN}^2}{Z_W}.$$

Problem 2. Find how many times (how many percent) is it possible to raise the power P_L dissipated in a load Z_L , if the level of matching of a load with a transmission line is raised from $K_{SW1} = 2,5$ to $K_{SW2} = 1,5$.

Methodical recommendations to the solution of a problem 2. As the power P_L dissipated in a load Z_L , is determined by expression

$$P_L = P_{L_{\max}} (1 - \Gamma_L^2),$$

and Γ_L (module of reflection coefficient) taking into account known K_{SW} on the basis of (2.14),

$$\frac{P_L}{P_{L\max}} = (1 - \Gamma_L^2) = \frac{4K_{SW}}{(K_{SW} + 1)^2}.$$

Then increase of power P_L dissipated in a load Z_L for the level of matching of a load with a transmission line raised from K_{SW1} to K_{SW2} where $K_{SW1} > K_{SW2}$, can be determined from following expression

$$\delta P_L = \frac{P_L \text{ (for } K_{SW2})}{P_L \text{ (for } K_{SW1})} = \frac{K_{SW2} (K_{SW1} + 1)^2}{K_{SW1} (K_{SW2} + 1)^2}.$$

Problem 3. Calculate parameters of matching device in the form of the quarter wave transformer for matching of a transmission line with a wave impedance $Z_W = 100 \Omega$ that is connected to a load $Z_L = 50 + i100 \Omega$. Plot the matching schema.

Problem 4. Calculate parameters of matching device in the form of Tatarinov's shunt for matching of a transmission line with a wave impedance $Z_W = 100 \Omega$ that is connected to a load $Z_L = 50 + i100 \Omega$. Plot the matching schema.

Methodical recommendations to the solution to problems 3 and 4. Among the parameters of matching device there are its length, wave impedance and place of insertion. The matching schema is similar to the schemas shown in fig. 2.15 and 2.16.

According to a statement of the problem 3 the matching device is quarter wave transformer, that is the transformer with length $l_{tr} = \lambda_w/4$, where λ_w is wave length of the microwave oscillation in a transmission line. As the quarter wave transformer provides matching of only active resistances it is necessary to insert it into the place where reactive component of the line full impedance is equal to zero, that is an extreme point of electric field distribution.

Positions of extremes in a transmission line is determined by expressions (2.10) and (2.11) which, in turn, are calculated on the basis of an argument (phase) of the reflection coefficient φ_L (2.19). From every possible values calculated on the basis of these expressions, one needs to choose the minimum positive value (less than $0,25\lambda_w$) that will correspond to the extreme nearest to the load or the place of quarter wave transformer insertion l_0/λ_w .

The wave impedance of the quarter wave transformer Z_W^{tr} is calculated on the basis of (2.25) or (2.26) depending on extreme type (a maximum or a minimum). The numerical value of K_{SW} in (2.25) and (2.26), can be calculated via the module of a reflection coefficient Γ_L (2.13) which, in turn, is calculated on the basis of (2.18).

According to a statement of the problem 4 the matching device in form of Tatarinov's shunt is used. The device in its design is a short-circuited piece of the similar transmission line with the same design parameters. Therefore one need to accept $Z_W^{sh} = Z_W$.

Due to the fact that Tatarinov's shunt possesses purely reactive input impedance (2.29) and is intended for neutralization of a reactive component of impedance of a transmission line it is necessary to insert it in a point where the active component of full transmission line admittance is equal to a transmission line original admittance.

Place of insertion of Tatarinov's shunt l_0/λ_w and its length l_{sh}/λ_w where λ_w is wave length of the microwave oscillations in a transmission line, can be calculated on the basis of expressions (2.31) and (2.33) accordingly.

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