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Odessa National Academy of Telecommunication named after A.S. Popov

Department of Telecommunication Theory named after A. Zuko

Ivaschenko P., Borschova L., Rozenvasser D.

TRANSMISSION OF INFORMATION IN TELECOMMUNICATION SYSTEMS

Module № 2

Education manual on telecommunication theory

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This education manual contains:

– main theoretical positions of Telecommunication Theory, chapter Transmission of information in telecommunication systems;

– methodical guidelines for fulfilling laboratory works;

– individual tasks with input data and methodical instructions for it performance and

– short English-Russian and Russian-English dictionaries on transmission of information in telecommunication systems.

The manual is intended for students training on a direction of Telecommunication studying the module 2 of Telecommunication theory.

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CONTENTS

1 INFORMATION CHARACTERISTICS OF MESSAGE SOURCES

1.1 Quantitative measure of the information

Transmission systems are created for transfer of the information. It is necessary to be able to calculate the quantity of the information, which is given out by a source and transferred by a channel. Also it is necessary to estimate limiting possibility of transmission channel for transfer of the information.

Therefore it is necessary to use some quantitative measure, which would enable to estimate objectively quantity of the information which message contains in. Such measure has been entered by K. Shannon in 1948.

The quantity of the information in the message a (a sign, a word, a phrase) is defined as:

$$I(a) = \log_2 \frac{1}{P(a)} = -\log_2 P(a), \tag{1.1}$$

where $P(a)$ – probability of the message a. The relation (1) is an axiom.

The probability of the message there is less, the more information contains in it.

The logarithmic function uses for realization of two obvious properties.

1. The message is beforehand known ($P(a) = 1$ – uncertainty is absent). Then the quantity of the information in the message a is equal to zero: $I(a) = \log_2 1 = 0$.

2. The source consistently chooses two independent messages a_j and a_k. The probability of such choice $P(a_j, a_k)$ is joint probability of events a_j and a_k

$$\begin{aligned} I(a_j, a_k) &= -\log_2(P(a_j, a_k)) = -\log_2(P(a_j)P(a_k)) = \\ &= -\log_2(P(a_j)) - \log_2(P(a_k)) = I(a_j) + I(a_k) \end{aligned} \tag{1.2}$$

Quantity of the information in two messages is equal to the sum of information quantities in each message (the measure is additive).

Unit of measure of information quantity is binary unit or bit.

Example 1. The probability of message a is equal to 1/16. Define the information quantity in this message.

Solution. $I(a) = -\log_2(1/16) = 4$ bit.

1.2 Models of discrete message sources

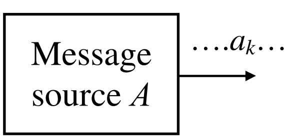

Figure 1 – Message source A

Let's consider a source of messages A (figure 1). The source gives out sequence of signs. The source chooses signs from the alphabet randomly $\{a_k\} = a_1, a_2, \ldots a_{M_A}$, where M_A – size of the source alphabet. For the source description it is necessary to specify probabilities of signs. There can be such cases.

1. Source without memory: the probability of given sign does not depend on signs what were before it and will be after it. In this case the source is given by

probabilities of signs $P(a_1)$, $P(a_2)$, ... $P(a_{M_A})$. Special case – signs is equiprobable $P(a_1) = P(a_2) = ... P(a_{M_A}) = 1/M_A$.

2. Source with memory: the probability of given sign depends on signs what were before it and will be after it. In this case a source is given by joint probabilities of signs.

For example, dependence only between two signs: $P(a_j, a_k) = P(a_j)\cdot P(a_k/a_j)$; dependence between three signs $P(a_j, a_k, a_l) = P(a_j)\cdot P(a_k/a_j)\cdot P(a_l/a_j, a_k)$, etc.

1.3 Entropy of an independent message source

Source is characterized by average quantity of the information, fall on one sign. The average quantity is called **source entropy**.

1. We shall consider a source without memory. It is described by probabilities of separate signs. If probabilities of signs are different, then signs contain different quantity of the information according to the formula (1.1)

$$I(a_k) = -\log_2 P(a_k).$$

Let's find source entropy

$$H(A) = \overline{I(a_k)} = -\sum_{k=1}^{M_A} P(a_k)\cdot\log_2 P(a_k). \tag{1.3}$$

Entropy properties:

1. Entropy is a real, limited and non-negative value. Property follows from a structure of expression for $H(A)$, and consequently that $0 \le P(a_k) \le 1$.

2. Entropy of determined messages source is equal to zero, $H(A) = 0$ because the probability of one of messages is equal to one and others are equal to zero.

3. Entropy is maximal, if all source messages are equiprobable. We shall prove it.

Let's consider a difference

$$\begin{aligned} H(A) - \log_2 M_A &= -\sum_{k=1}^{M_A} P(a_k)\cdot\log_2 P(a_k) - \log_2 M_A = -\sum_{k=1}^{M_A} P(a_k)\cdot\log_2 M_A P(a_k) = \\ &= \log_2 e \sum_{k=1}^{K} P(a_k)\cdot\ln\frac{1}{M_A P(a_k)}. \end{aligned} \tag{1.4}$$

Let's take advantage of a known relation

$$\ln x \le x - 1, \tag{1.5}$$

which is correct for any positive x. Then

$$\begin{aligned} H(A) - \log_2 M_A &\le \log_2 e \sum_{k=1}^{M_A} P(a_k)\cdot\left[\frac{1}{M_A P(a_k)} - 1\right] = \\ &= \log_2\left[\sum_{k=1}^{M_A}\frac{P(a_k)}{M_A P(a_k)} - \sum_{k=1}^{M_A} P(a_k)\right] = \log_2[1-1] = 0, \end{aligned} \tag{1.6}$$

equality takes place only at $M_A P(a_k) = 1$, so:

$$P(a_1) = P(a_2) = ...= P(a_K) = 1/M_A$$

and then

$$H_{\max}(A) = -\frac{1}{M_A}\sum_{k=1}^{M_A}\log_2\left(\frac{1}{M_A}\right) = \log_2 M_A . \qquad (1.7)$$

Apparently from last expression, in case of equiprobable messages entropy increases with increasing of source alphabet size (growth of signs number). At not equiprobable signs entropy, accordingly, decreases.

For example, the source A of discrete independent messages uses 4 signs with probabilities $P(a_1) = 0{,}7$; $P(a_2) = P(a_3) = P(a_4) = 0{,}1$. Let's define source entropy.

$$H(A) = -\sum_{k=1}^{K} P(a_k)\log_2 P(a_k) = -\,0{,}7\cdot\log_2 0{,}7 - 3\cdot 0{,}1\cdot\log_2 0{,}1 = 1{,}36 \text{ bit}.$$

The maximal value of source entropy for alphabet size M_A is

$$H_{\max}(A) = \log_2 M_A = \log_2 4 = 2 \text{ bit}.$$

Properties of source entropy can be easy calculated in case of a binary source ($M_A = 2$). Let probabilities of signs $P(a_1) = p$, and $P(a_2) = 1 - p$. Then binary source entropy will be written down

$$H(A) = -P(a_1)\log_2 P(a_1) - P(a_2)\log_2 P(a_2) = -p\log_2 p - (1-p)\log_2(1-p). \qquad (1.8)$$

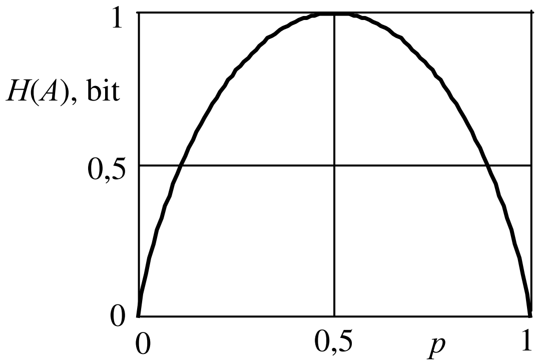

Figure 2 – Binary source entropy

On figure 2 dependence (1.8) is plotted. From figure it is visible, that entropy is equal to zero at $P(a_1) = 0$; $P(a_2) = 1$ or $P(a_1) = 1$; $P(a_2) = 0$; the maximum of entropy takes place, when $P(a_1) = P(a_2) = 0.5$, and the maximal value is equal 1 bit, as well as calculated under the formula (1.7). If probabilities of signs different and final then entropy accepts values between zero and one.

1.4 Entropy of a dependent message source

Strong statistical connection between signs appears on the source output in actual practice.

So, in texts, probabilities of separate letters depend on what letters preceded them. For example, let's consider the English sensible text. The letter "P" has appeared. The probability of that following will be "A", much more, than probability of the letter "H". At transfer of images observes the similar situation – the adjacent elements of the image have usually almost identical brightness and color.

For definition of source entropy when signs are statistically connected, it is necessary to take into account conditional probabilities of separate signs. Let two signs of the message following one after another are statistically connected. Using formula (1.1), we shall define quantity of the information in sign a_k what appeared after (already known) sign received earlier a_j

$$i(a_k/a_j) = -\log_2 P(a_k/a_j). \qquad (1.9)$$

In this case entropy of dependent messages source is calculated by averaging on indexes k and j:

$$H_2(A) = \overline{I(a_k/a_j)} = -\sum_{j=1}^{M_A}\sum_{k=1}^{M_A} P(a_j, a_k)\log_2 P(a_k/a_j) = \\ = -\sum_{j=1}^{M_A} P(a_j)\sum_{k=1}^{M_A} P(a_k/a_j)\log_2 P(a_k/a_j). \qquad (1.10)$$

Calculation becomes complicated. Important that, a source entropy decreases if connection between signs present

$$H_0(A) \geq H_1(A) \geq H_2(A) \geq \ldots H_n(A), \qquad (1.11)$$

The expression specifies how many signs have statistical dependence.

So, for the Russian text $H_0(A)$ = 5 bit, $H_8(A)$ = 2 bit.

1.5 Redundancy of a message source

Redundancy of a messages source is a property of a source to give out information by greater number of symbols, than it could be possible. Quantitatively redundancy is defined by coefficient of redundancy on the formula

$$\chi_r = \frac{H_{max}(A) - H(A)}{H_{max}(A)}. \qquad (1.12)$$

The formula (1.12) can be treated so: redundancy shows relative underloading of signs by information on the average. Properties of redundancy:

1. Positive property is the opportunity to correct the mistakes arising at message transfer.
2. Negative property is loading of a communication channel or the storage device superfluous symbol.

Example. Find the coefficient of redundancy of the source:

$H(A)$ = 1,36 bit, $H_{max}(A)$ = 2 bit.

Decision. χ_r = (2 – 1,36)/2 = 0,32.

1.6 Source rate

Source rate is quantity of the information which are given out by a source on the average per 1 second

$$R_s = \frac{H(A)}{T_{symb}}, \qquad (1.13)$$

where T_{sign} – average time, spent by a source on delivery with one symbol.

1.7 Joint entropy and mutual information

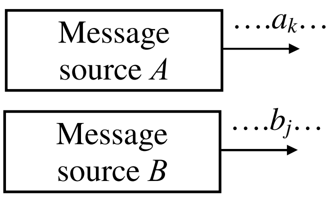

Figure 3 – Two message sources

Let's consider two message sources A and B (figure 3). Joint entropy is average quantity of information which contains in common appearing signs a_k and b_j

$$H(A,B) = -\sum_{k=1}^{M_A}\sum_{j=1}^{M_B} P(a_k,b_j)\log_2 P(a_k,b_j). \qquad (1.14)$$

It is is possible to calculate joint entropy:

$$H(A,B) = H(A) + H(B/A) = H(B) + H(A/B), \quad (1.15)$$

where $H(B/A)$ – conditional entropy of source B, the average quantity of information in one sign of source B provided that the message of source A is known. The similar definition of conditional entropy $H(A/B)$. If signs of sources A and B are independent, then $H(B/A) = H(B)$, $H(A/B) = H(A)$.

For sources of messages A and B the mutual information is entered. It is average quantity of the same information, which contains in signs of sources messages A and B

$$I(A,B) = I(B,A) = H(A) - H(A/B) = H(B) - H(B/A). \qquad (1.16)$$

If signs of sources A and B are independent, then $I(A,B) = I(B,A) = 0$. Mutual information $I(A,B)$ also called as the information from A to B.

1.8 Models of continuous message sources

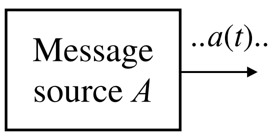

Figure 4 – Message source A

Let's consider a continuous messages source A (figure 4). The source gives out the continuous message $a(t)$. The message will be transformed by different transducers to a baseband telecommunication (analogue) signal $b(t)$: $b(t) = k \cdot a(t)$, where k is a factor of proportionality. We shall consider further, that the source gives out a signal $b(t)$.

For the decision of information tasks it is necessary to set probabilistic characteristics. It is enough to know a probability density $p(b)$.

1.9 Differential entropy of a continuous signal

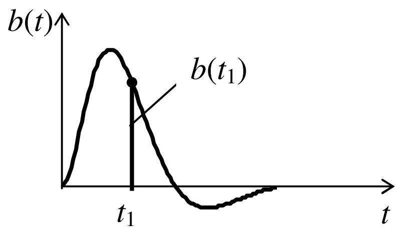

Figure 5 – Continuous signal

There is a signal on the source output (figure 5). We shall find quantity of information in one sample $b(t_1)$

$$I(b(t_1)) = \log_2 \frac{1}{P(b(t_1))}.$$

As $P(b(t_1)) \to 0$, then $I(b(t_1)) \to \infty$.

Let's execute quantization of sample (digitization on a level). Then sample will accept values b_1, b_2, $\ldots b_L$, where L – number of quantization levels.

Probabilities of these values are finite

$$P(b_k) = p(b_k)\Delta b .$$

Entropy of samples

$$H(B) = \lim_{\Delta b \to 0} -\sum_{k=1}^{L} p(b_k)\Delta b \log_2 (p(b_k)\Delta b) = \ldots = -\int_{-\infty}^{\infty} p(b)\log_2 p(b) - \lim_{\Delta b \to 0} \log_2 \Delta b . \quad (1.17)$$

The first component names as differential entropy

$$h(B) = -\int_{-\infty}^{\infty} p(b)\log_2 p(b). \quad (1.18)$$

The second component $-\lim_{\Delta b \to 0} \log_2 \Delta b \to \infty$ (tends to infinity).

Thus, the quantity of the information in one sample tends to infinity.

For comparison of different messages differential entropy is used. The second component for all signals is the same.

Differential entropy is maximal, if the signal has normal probability distribution

$$p(b) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{b^2}{2\sigma^2}},$$

where σ^2 – dispersion of signal $b(t)$.

After substitution we receive

$$h(B) = \log_2 \sqrt{2\pi e \sigma^2} \text{, bit/sample.} \quad (1.19)$$

1.10 Epsilon-entropy of a continuous signal

Any continuous signals suppose the approximate representation $\hat{b}(t)$.

There is an error of approximate representation $\varepsilon(t) = \hat{b}(t) - b(t)$. Quantitatively the size of error is described by its average square $\overline{\varepsilon^2(t)}$.

Epsilon-entropy $H_\varepsilon(B)$ is minimum average quantity of mutual information between $b(t)$ and $\hat{b}(t)$ in one sample at the given allowable error of its approximate representation $\overline{\varepsilon^2(t)} \le \varepsilon_0^2$:

$$H_\varepsilon(B) = \min\{h(B) - h(B/\hat{B})\} = h(B) - \max h(\varepsilon). \quad (1.20)$$

where $h(\varepsilon)$ – differential entropy of error $\varepsilon(t)$.

The maximal value $h(\varepsilon)$ takes place at the normal distribution $\varepsilon(t)$, and formula for calculating of epsilon entropy looks like

$$H_\varepsilon(B) = h(B) - \log_2 \sqrt{2\pi e \sigma^2}, \quad (1.21)$$

where σ^2 – dispersion of error $\varepsilon(t)$.

1.11 Redundancy of a continuous message source

The redundancy factor of continuous message source can be calculated according to the formula (1.12) at substitution of corresponding values of differential entropy $h(B)$ and $h_{max}(B)$.

$$\chi_r = \frac{h_{max}(B) - h(B)}{h_{max}(B)}. \tag{1.22}$$

If the signal has normal distribution, then $\chi_r = 0$.

1.12 Continuous message source rate

Epsilon rate R_s of a continuous message source can be calculated according to the formula (1.13) in which it is necessary to substitute values of epsilon entropy $H_\varepsilon(B)$, and sampling interval $T_s = 1/(2F_{max})$

$$R_s = 2F_{max} h(B). \tag{1.23}$$

2 EFFECTIVE CODING OF MESSAGES

2.1 Problem of message coding

The problem of message coding consists in representation of messages by a digital signal (binary symbols). Binary symbols are convenient for transmitting and storing.

At first we shall consider coding of discrete messages. Message coding is carried out on the basis of the certain code. The code is a rule or the table, according to which each sign in the message put in conformity the code word (a set of binary symbols). As a result of coding we receive a baseband digital signal. Inverse transformation of baseband digital signals to messages called decoding, which is carried out by the decoder on the basis of the same code.

Inclusion of the source coder is shown on figure 6: M_A – size of message source alphabet; m – the code base, as a rule, equal 2.

Coding performs without information losses. Therefore source rate is same, on the source output, and on the coder output. On figure 6 inclusion of the source coder is shown. The basic characteristic of a digital signal is its rate R.

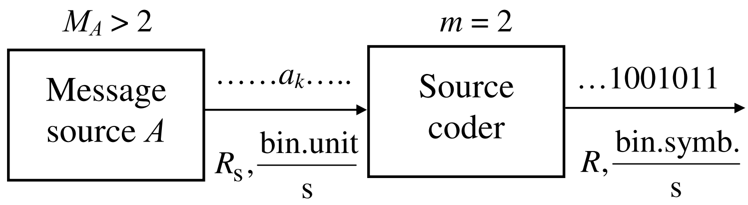

Figure 6 – Inclusion of the source coder

If lengths of code words are identical and equal n the code is called uniform (fixed length) or primitive. The length of a binary code depends on size of the source alphabet and is defined as:

$$n \geq \log_2 M_A, \qquad (2.1)$$

Length of code words can differ at non-uniform (variable-length) codes. At construction of a non-uniform code it is necessary take into account probabilities of coded signs: to those signs which meet more often, give shorter code words and vice versa. Therefore non-uniform codes name statistical. Example of such code is the code (alphabet) Morse (code Morse was created near 170 years ago). Morse has made his code intuitively. Below we shall formulate rigorous methods of non-uniform codes construction.

2.2 Primitive codes for discrete messages

Examples of a primitive code are the following basic codes: ITA-2, ASCII, EBCDIC. Feature of primitive coding is that if the source message has redundancy then redundancy does not reduce by the coder, redundancy can even increase.

Code ITA-2 (International Telegraph Alphabet № 2) has length n = 5, that is $M \leq 32$. How it is possible to code Latin and Russian letters, Arabian digits and moreover some signs? Practically, we have three code tables, and before the beginning of coding the register, what determines what table the coder and, accordingly, the decoder should use, is underlined. Before transition to another table it is necessary to replace the register.

Codes ASCII (American Standard Code Information Interchange) and EBCDIC (Extended Binary-Coded-Decimal Interchange Code) has length $n = 8$, there is each sign is coded with byte.

From the point of view of the information theory the use of primitive codes does not demand the decision of any problems.

2.3 Shannon theorem for the channel without noise

The Shannon theorem of coding for the channel without noise (for a source) approves, that the average length $\bar{n}$ of code words (binary sequences) can be arbitrary closely to the source entropy:

$$\bar{n} \geq H(A) + \varepsilon, \tag{2.2}$$

where $H(A)$ is message source entropy;

ε – arbitrary small value.

From entropy properties for a binary source (on the coder output) follow, that entropy cannot exceed 1 binary unit. Hence, the average length of a code word $\bar{n}$ cannot be less then entropy.

Let's analyse, can an equality $\bar{n} = H(A)$ be there?

Let's define a source rate on coder output, considering, that coded signs independent

$$R_\text{s} = \frac{H(A)}{T_\text{sign}} = \frac{H(A)}{T_\text{b}\bar{n}} = \frac{-\sum_{k=1}^{M} P(a_k)\log_2 P(a_k)}{T_\text{b}\sum_{k=1}^{M} P(a_k)n_k}. \tag{2.3}$$

We admit, that at coding of sign a_k length of code word is:

$$n_k = -\log_2 P(a_k) \text{ for all } k. \tag{2.4}$$

Then the sums in numerator and denominator will be equal and the formula (2.3) pass to next:

$$R_\text{s} = \frac{1}{T_\text{b}} = R. \tag{2.5}$$

That is, each binary symbol transfers one binary unit of the information and equality is carried out $\bar{n} = H(A)$. Really equality (2.4) cannot be executed exactly for all k and then $\bar{n}$ come close to $H(A)$. For reduction ε in the ratio (2.2) it is necessary to code the big blocks of signs. It means to pass to the integrated alphabet.

2.4 Principles of discrete messages effective coding

Coding at which the average length of code word is minimized is called effective (economical) coding. The same that at the fixed source rate R_s the rate of a digital signal on the coder output is minimized. Effective coding called also compression of the information.

For realization of effective coding it is necessary to take off redundancy at coding. The reasons of redundancy are: dependence between signs of source and not equiprobable signs.

Accordingly, at coding 2 stages are carried out.

– Dependence between signs of a source is eliminated at coding by transition to the integrated signs, which will be independent. For example, to pass from coding letters to coding words or phrases.

– The integrated independent signs are coded by one of the developed effective codes: Shannon-Fano or Huffman.

For estimation of compression methods efficiency the parameter a degree of compression is used:

$$\mu = n/\overline{n}\,, \tag{2.6}$$

where n – length of code word at uniform coding.

2.5 Shannon-Fano code

Optimum algorithms of compression of discrete messages with independent signs are Shannon-Fano and Huffman algorithms. These algorithms have much common and provide practically identical degree of compression. Necessary operating condition of these algorithms is that signs probabilities should be known $P(a_0)$, $P(a_1)$, … The principle of coding consists that to more probable signs shorter words are appropriated, and to less probable signs longer words are appropriated. Besides probabilities of symbols 1 and 0 on coder output any way should be equal or almost equal.

The Shannon-Fano algorithm consists in following. Signs write down in decreasing order their probabilities. Then signs share on two groups so that the sums of signs probabilities of each group were approximately identical. To signs from first group, as the first symbol of a non-uniform code, the zero is assigned, and to symbols of the second group the unit is assigned. Each of the received groups, if it contains more than one sign, shares for two equiprobable groups and a rule of coding repeats. This process proceeds until in each group one coding sign will stay.

Let the source, which is giving out the messages with the help of eight signs, is given. Probabilities of these signs: $P(a_0)$ = 0,05; $P(a_1)$ = 0,3; $P(a_2)$ = 0,2; $P(a_3)$ = 0,1; $P(a_4)$ = 0,06; $P(a_5)$ = 0,15; $P(a_6)$ = 0,11; $P(a_7)$ = 0,03.

Source entropy equal to:

$$H(A) = -\sum_{k=1}^{M} P(a_k)\cdot \log_2 P(a_k) = 2{,}524 \text{ binary units.}$$

In tab. 1 the non-uniform code made on Shannon-Fano algorithm is shown. Let's calculate average length of a code word

$$\overline{n} = -\sum_{k=1}^{M} P(a_k)\cdot n_k = 2{,}72.$$

Though for separate symbols length of a code word is equals 5, the average length of a code word $\bar{n}$ = 2,72. In case of uniform coding n = 3. Degree of compression in this case μ = 1,103. Comparison source entropy and average length of a code word shows, that in this case the maximal degree of compression is not achieved.

Table 1 – Coding on the Shannon-Fano algorithm

Sign a_k	Probability $P(a_k)$	Appropriated binary symbols				
a_1	0,3	**0**	**0**			
a_2	0,2	**0**	**1**			
a_5	0,15	**1**	**0**	**0**		
a_6	0,11	**1**	**0**	**1**		
a_3	0,1	**1**	**1**	**0**		
a_4	0,06	**1**	**1**	**1**	**0**	
a_0	0,05	**1**	**1**	**1**	**1**	**0**
a_7	0,03	**1**	**1**	**1**	**1**	**1**

2.6 Huffman code

In case of the Huffman coding algorithm the coding signs write down in decreasing order their probabilities. If some signs have identical probabilities, they are placed in any order. Then it is necessary to construct the probability tree: choose two signs with the least probabilities and form the first branch of the tree. The chosen signs unite into "intermediate" sign with probability, equal to the sum of probabilities of the chosen signs. Then among the staying signs (together with intermediate sign) again find two signs with the least probabilities and do the same as on the first step. This procedure carries out until all signs from source alphabet and intermediate signs will be used and the root of tree with probability equal 1 will be received.

“Movement” on a tree from root to corresponding symbol carries out getting of code words. Passage through a branch means addition of a binary symbol to code word: if "movement" through upper branch then unit is added and if down branch then zero is added.

Let's consider an example of signs coding, the alphabet consist of 7 signs. In table 2 signs are resulted and their probabilities are point. On figure 7 construction of probability tree is shown. For this purpose signs are located from top to down in decreasing order their probabilities. Procedure of probability tree construction described above. The received code words are shown in table 2.

Entropy is calculated : $H(A)$ = 2,524 binary unit.

Let's calculate average length of a code word

Table 2 – Huffman code

Sign a_k	$P(a_k)$	Code word
a_0	0,05	**10001**
a_1	0,3	**11**
a_2	0,2	**01**
a_3	0,1	**000**
a_4	0,06	**1001**
a_5	0,15	**101**
a_6	0,11	**001**
a_7	0,03	**10000**

$$\bar{n} = -\sum_{k=1}^{M} P(a_k) \cdot n_k = 2{,}72.$$

Average length of the received code the same as for Shannon-Fano code.

According to Shannon theorem of source coding the average length of code word cannot be less source entropy. For our example

$$\bar{n} = 2{,}72 > H(A) = 2{,}524 \text{ bit.}$$

These values differ only on 7%.

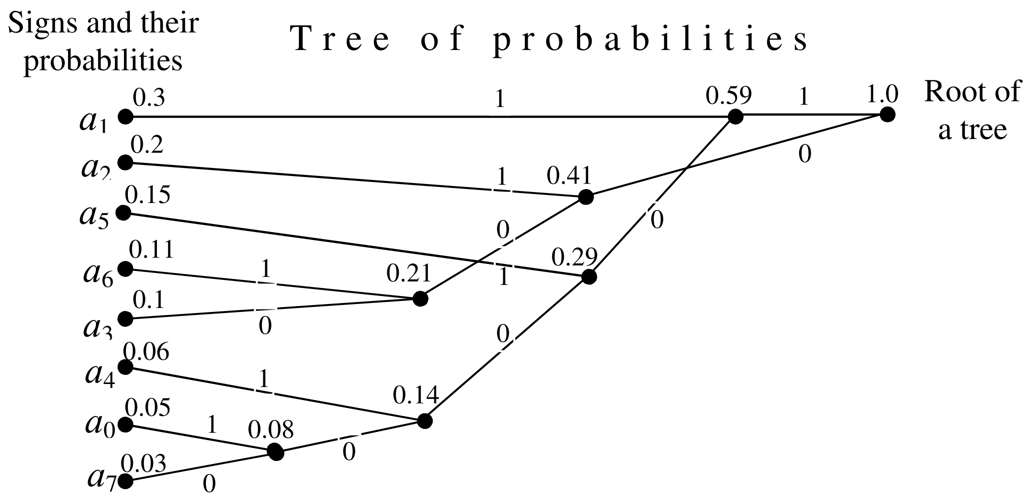

Figure 7 – Coding on Huffman algorithm

2.7 Application of discrete messages effective coding

It is necessary to note, that Huffman and Shannon-Fano codes are **prefix** codes. The codes are called prefix codes when there are no demanding separating symbols between code words. From tables of Huffman and Shannon-Fano codes it is visible, that any of short code words is not the beginning of longer word.

Let's code sequence of signs… a_1 a_6 a_3 a_1 a_2 a_0 … with Huffman code

… 1 1 0 0 1 0 0 0 1 1 0 1 1 0 0 0 1 …

We use the same table for decoding and we shall receive

… a_1 a_6 a_3 a_1 a_2 a_0 …

If a code uniform (primitive) separating symbols between code words are required. For example, $M = 4$

a_i	a_0	a_1	a_2	a_3
Code word	00	01	10	11

Let's code sequence of signs… a_1 a_3 a_2 a_1 a_2 a_0 … with the mentioned code

… 0 1 1 1 1 0 0 1 1 0 0 0 … It is necessary to know boundaries of code words for correct decoding. For this purpose enter a separating symbols.

… 1 0 1 1 1 1 1 1 0 1 0 1 1 1 0 1 0 0 1 …

Separating symbols are redundant, additional resource of a communication channel is spent for their transmission.

Huffman codes are used in some telecommunication technologies, for example, data compression in Rec. V.42, coding of images by method MPEG (Motion Picture Expert Group).

The universal algorithm of compression Ziv-Lempel-Whelch is used in modern computer archivers. This algorithm is updated Huffman algorithm. For some types of files the factor of compression can achieve 5–10.

2.8 Digital methods of analog signals transmission

Methods of digital transmission of analog signals are widely used in modern telecommunications:

– the analog signal will be transformed in digital, to the sequence of binary symbols;
– the digital signal is transmitted by a digital communication channel;
– from digital signal the analog signal is recovered.

Any method of digital transmission is characterized by rate of digital signal R (bit per second) and accuracy of transmission – the signal/quantization noise ratio ρ_q. The problem is to satisfy the requirement to the ratio ρ_q at minimal value R. There were developed a lot of digital methods of transmission. The elementary methods among them are pulse code modulation (PCM)[1].

2.9 Discretization of analog signals on time

At any method of digital transmission the analog signal $b(t)$, first of all, will be transformed to a discrete signal which represents itself sequence of samples $b(kT_s)$, taken through an sampling interval $T_s \le 1/(2F_{max})$, where F_{max} is the maximal frequency of a signal spectrum $b(t)$. This transformation names sampling of a signal on time, and the device for its realization names sampler. Sampling frequency should be not less the double frequency F_{max}:

$$f_s = 1/T_s \ge 2F_{max}.$$

According to Kotelnikov theorem, performance of this ratio guarantees an opportunity of exact recovery of an analog signal on samples. Such recovery is carried out by LPF with a cut frequency F_{max} (figure 8).

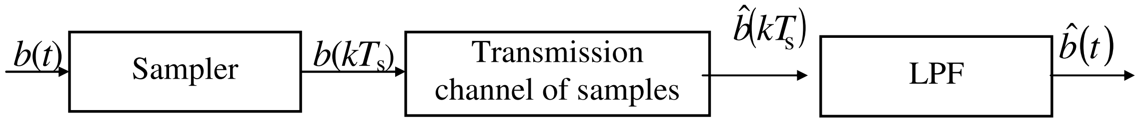

Figure 8 – Transmission of an analog signal by samples

All methods of digital transmission of analog signals differ in the ways of representation of discrete signals by digital signals. The converter of samples in a

[1] Despite of presence of a word "modulation", these methods of transmission have relation neither to analog, nor to digital modulation.

digital signal is called as the coder of digital transmission system (DTS), and the converter of a digital signal in samples is called as decoder DTS.

2.10 Methods of pulse code modulation

Feature of these methods is that each sample is represented by a digital signal independently of other samples. Methods differ between themselves a used code. The diagram that describes transmission of samples at PCM is represented on figure 9.

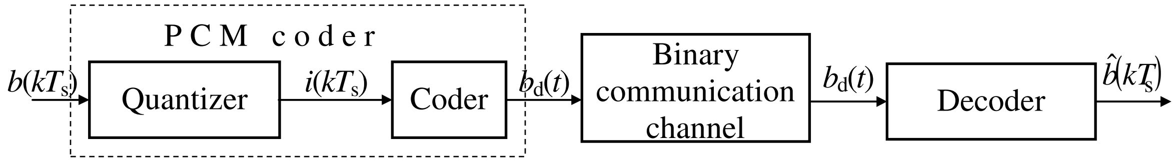

Figure 9 – PCM coder and decoder

Basic parameter of quantizer is number of quantization levels L. At uniform quantization a range of values b from $-b_{max}$ to b_{max} is broken on $L - 1$ intervals in size

$$\Delta b = 2b_{max} /(L - 1),$$

which is called as step of quantization. On figure 10 shown breakdown at $L = 8$. Discrete values b_i correspond to the middle of intervals. The index i accepts values 0, ±1, ±2, …, ±0,5L – 1. Discrete values are determined $b_i = i \cdot \Delta b$. At quantization each sample $b(kT_s)$ is approximated to the nearest discrete value b_i, and on an quantizer output the integer $i(kT_s)$ acts. Representation of sample $b(kT_s)$ by discrete value b_i brings an error

$$\varepsilon_q(kT_s) = i(kT_s)\cdot\Delta b - b(kT_s),$$

which is called as quantization noise.

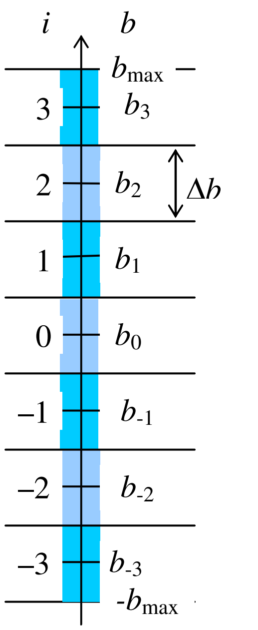

Figure 10 – To the explanation of quantization

In the coder included in PCM coder (figure 9), numbers $i(kT_s)$ are represented by the given binary code. Length of a code

$$n = \log_2 L.$$

The digital signal on an coder output $b_d(t)$ has rate
$R = n \cdot f_s$.

The decoder from a digital signal $b_d(t)$ forms numbers $i(kT_д)$ on which are recovered quantizing samples: $b_q(kT_s) = i(kT_s)\Delta b$. They are recovered samples of a transmitted analog signal $\hat{b}(kT_s) = b_q(kT_s)$. From formula follows, that samples are recovered with errors $\varepsilon_q(kT_s)$.

The average square of an quantization error (average power of quantization noise) is determined by a quantization step

$$\overline{\varepsilon_q^2} = \frac{(\Delta b)^2}{12}$$

and the ratio signal/quantization noise

$$\rho_q = \frac{P_b}{\overline{\varepsilon_q^2}} = \frac{3(L-1)^2}{K_A^2}$$

where K_A is an amplitude factor of an analog signal.

The considered transformation of an analog signal to a digital signal with uniform quantization step called as analog-digital conversion (ADC); return transformation is called as digit-analog conversion (DAC). ADC and DAC diagrams are shown on figure 11.

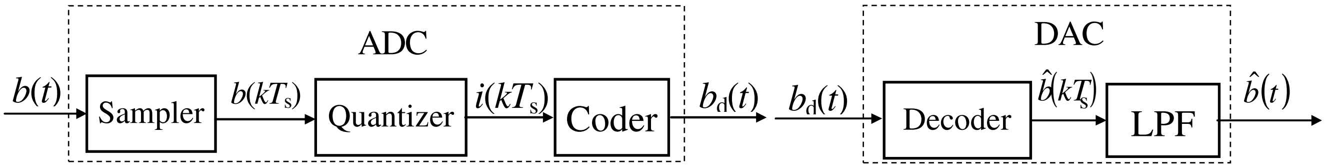

Figure 11 – ADC and DAC

Method PCM with non-uniform quantization are widely used: in the domain of great values $|b|$ steps of quantization big and on the contrary. It is equivalent to nonlinear transformation of samples with the subsequent uniform quantization (figure 12). Characteristics of nonlinearity are described by the A-law or μ-law. Due to such transformation decreases K_A and grows ρ_q at constant number L, or, keeping value ρ_q, it is possible to reduce number L, length of a code n and a digital signal rate R. Such method of PCM is standardized the Rec. G.711. The sampling frequency f_s = 8 kHz, the signal rate R = 64 kbit/s.

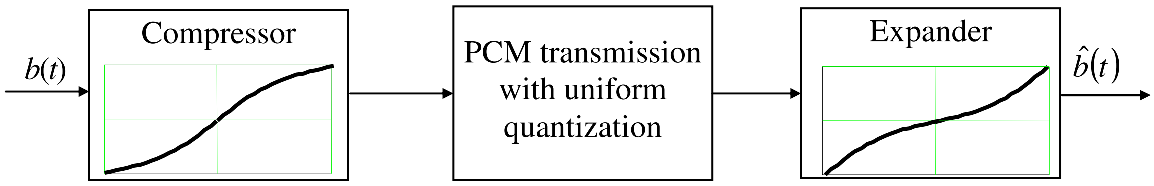

Figure 12 – PCM transmission with companding

2.11 Coding of analog signals with prediction

Samples of real analog signal are correlated. This fact allows to predict with any accuracy value of the next sample of a signal on values of the previous samples. In the coder of transmission system with a prediction (figure 8) the error of a prediction is calculated

$$d(kT_s) = b(kT_s) - \tilde{b}(kT_s),$$

where $b(kT_s)$ – the sample of an analog signal acting from sampler;

$\tilde{b}(kT_s)$ – the predicted sample generated by the predictor on basis N of the previous samples $b((k-1)T_s),\ b((k-2)T_s),\ \cdots b((k-N)T_s)$.

The error of a prediction is transferred on a communication channel by a digital signal. Therefore in the circuit of the coder of system (figure 13) quantizer and the coder of the prediction error samples are available. The decoder of samples recovers samples of a prediction error; in the system decoder there is precisely same predictor, as in system coder; the predicted samples develops with the transferred sample of a prediction error and, thus, sample of a transmitted analog signal are recovered.

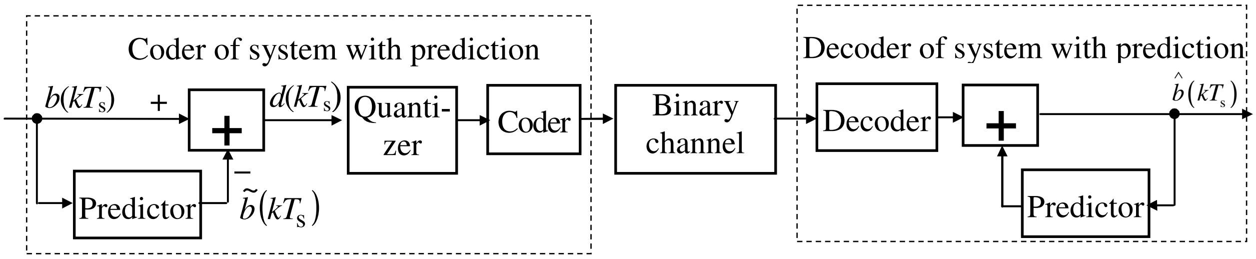

Figure 13 – Coder and decoder of system with prediction

Peak-to-peak of a discrete signal $d(kT_s)$ smaller, than peak-to-peak of a signal $b(kT_s)$, therefore number of quantization levels L at a constant quantization step Δd will be smaller, than transmission of samples $b(kT_s)$ by method PCM. Reduction of a quantization levels number reduces length of a code n and a digital signal rate $R = n \cdot f_s$. Or, at constant number of quantization levels L the step of quantization decreases $\Delta d = (d_{max} - d_{min}) / L$, the power of quantization noise decreases $\overline{\varepsilon_q^2} = \Delta d^2/12$, the ratio signal/quantization noise grows ρ_q.

2.12 Methods of differential PCM

In different variants of DPCM method the number of samples N on the basis of which predict sample are defined, is in limits from 1 up to 6. Predictor at $N \geq 2$ is carried out under the circuit of not recursive filter. At case $N = 1$ predicting sample $\tilde{b}(kT_s)$ is the previous sample $b((k-1)T_s)$.

Circuits of the DPCM coder and decoder, used in the real equipment, are resulted on figure 14. In the coder prediction error acts on quantizer, similar quantizer systems with PCM, then the quantized error $d_q(kT_s)$ is transferred by a digital signal on a communication channel. Predictors in the coder and the decoder are completely identical.

As opposed to circuit resulted on figure 13, the predictor in the coder is included in a loop of a feedback. Therefore predicting samples $\tilde{b}(kT_s)$ in the circuit of the coder and in the circuit of the decoder are developed from the same samples $\hat{b}(kT_s)$ (if in a channel there were no errors by transfer).

In the decoder the predictor is included in circuits of a feedback and consequently at decoding quantization noise can collect. Let's define a quantization error at DPCM under the circuit figure 14

$$\varepsilon_q(kT_s) = \hat{b}(kT_s) - b(kT_s) = [\tilde{b}(kT_s) + d_q(kT_s)] - [\tilde{b}(kT_s) + d(kT_s)] = d_q(kT_s) - d(kT_s).$$

From last ratio it is visible, that, due to inclusion of the predictor in the coder in a circuit of a feedback, the quantization error is defined only by quantizer parameters, and there is no effect of accumulation of quantization noise in the decoder.

2.13 Adaptive DPCM

Methods of adaptive DPCM (ADPCM) are widely applied. Adaptive parts of ADPCM coder are:

– predictor with N = 4…6 – its coefficients are automatically adjusted so that the dispersion of a signal $d(kT_s)$ was minimized, coefficients of the predictor are transferred on a communication channel, that in the predictor in decoder the same coefficients, as well as in the predictor of the coder were used;

– quantizer – peak-to-peak of its characteristic (d_{max}, d_{min}) and accordingly a quantization step changes in conformity to peak-to-peak of a current signal $d(kT_s)$, data on a quantization step are transferred by a channel, that in the decoder the quantization step same, as well as in quantizer was set.

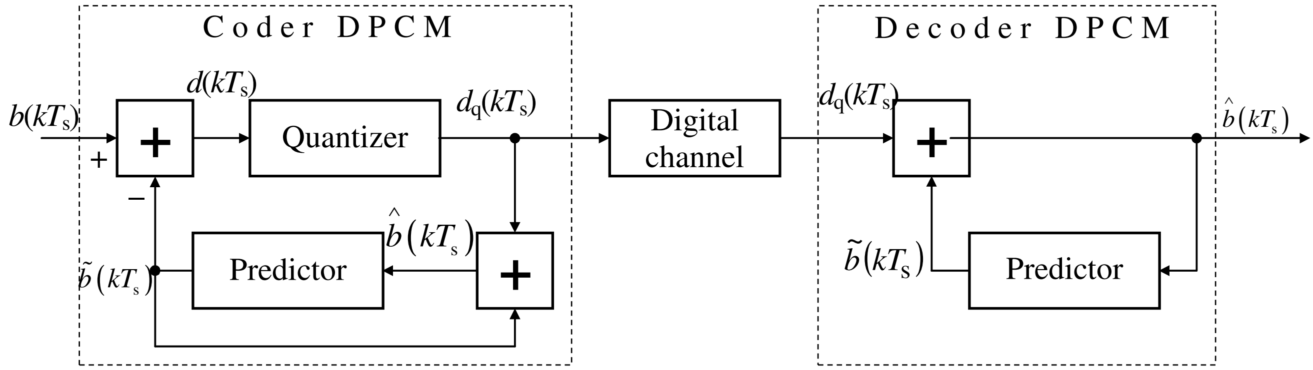

Figure 14 – Coder and decoder of system with DPCM

2.14 Methods of delta modulation (DM)

Methods of DM concern to methods of transfer with a prediction. Methods of DM differ from PCM and DPCM that two-level quantizer ($L = 2$) are used. It becomes possible if the sampling frequency greater, than $2F_{max}$. Then adjacent samples from the sampler differ a little. On figure 15 circuits of coder and decoder, explaining one of DM methods, are resulted.

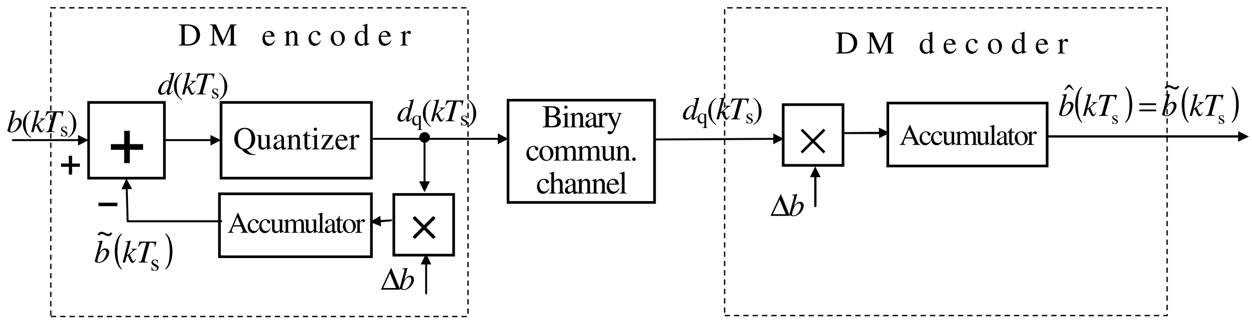

Figure 15 – DM encoder and decoder

The error of a prediction is calculated the same as at DPCM,

$$d(kT_s) = b(kT_s) - \tilde{b}(kT_s),$$

The error of a prediction is calculated the same as at DPCM,

$$d(kT_s) = b(kT_s) - \tilde{b}(kT_s),$$

and the predicted sample grows out works of the accumulator

$$\tilde{b}(kT_s) = \sum_{i=0}^{k-1} d_q(kT_s) \cdot \Delta b,$$

where Δb is coefficient;

$$d_q(kT_s) = \begin{cases} +1, & \text{if } d(kT_s) \geq 0, \\ -1, & \text{if } d(kT_s) < 0 \end{cases} -$$

Error of a prediction is quantized on two levels which are transferred by a binary communication channel.

The described method of coding is illustrated by time diagrams on figure 11. Here the predicted signal and a quantized prediction error signal are submitted by signals of continuous time. It is visible, that the predicted signal $\tilde{b}(t)$ "traces" changes of an input signal. The essence of factor Δb follows from figure – it is a step of quantization, as with this step a signal $\tilde{b}(t)$ quantizes. In figure it is visible two domains:

1) domain where are observed distortion of an overload on an inclination – the predicted signal $\tilde{b}(t)$ has not trace changes of an input signal;

2) domain where subdivision noise is observed – at a constant input signal the predicted signal changes in peak-to-peak amplitude Δb.

Clearly, that for reduction of the first effect it is necessary to increase a step of quantization, and for reduction of the second effect – to reduce a step of quantization. It is obvious, that there is an optimum step of quantization at which the total effect from an overload on an inclination and noise of subdivision on realizations of a signal $b(t)$ is minimized.

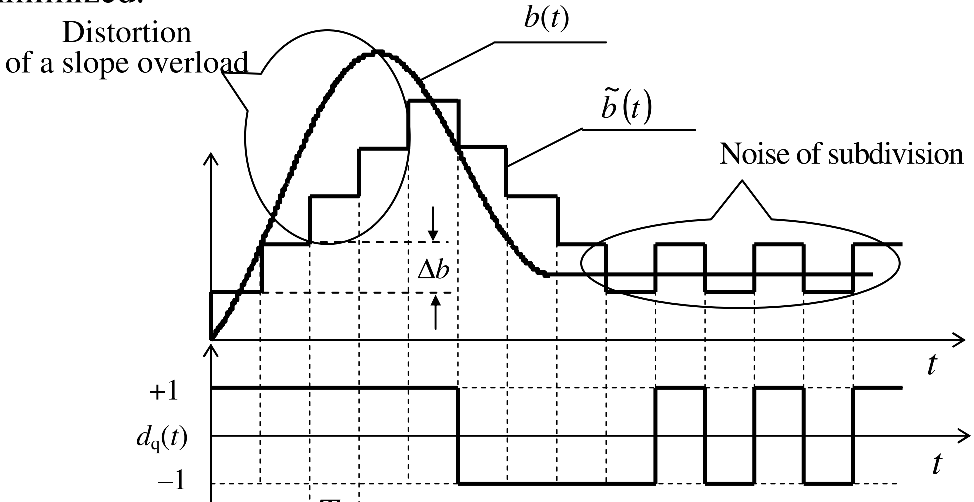

Figure 16 – Illustration of work of the DM coder

Work of the decoder of DM (figure 16) is contained to calculation of predicted samples of the signal under the formula for $\tilde{b}(kT_s)$.

It is possible to formulate features of DM methods:

- the sampling frequency f_s (figure 10) in some times is more, than $2F_{max}$;
- as quantizer is two-level so the code has length $n = 1$, and $R = f_s$;
- as $n = 1$, so necessity of the decoder synchronization disappears.

2.15 Adaptive DM (ADM)

At adaptive delta - modulation (ADM) the quantization step can change. It is carried out as follows. On a coder output the analyzer of sequence of binary symbols is connected. If there was a sequence 111 or 000, the step of quantization increases to reduce distortions of an overload on an inclination. If there was a sequence 101 or 010, the step of quantization decreases to reduce distortions from noise of subdivision.

The similar analyzer is connected on a decoder input and in the same way the step of quantization in the decoder changes.

2.16 Conclusion

The effective encoding of continuous signals is removed redundancy which is conditioned by statistical connection between samples (by correlation of samples). The effective encoding of continuous signals is based on the followings methods:

- static images compression (photo, fax) is subsampling (diminishing of frequency of sampling of color components), discrete Fourier cosine-transform, JPEG, GIF algorithms and other, vectorial compression (all curves on image are described by mathematical expressions);

- movable images compression (video) is methods of prediction (in the first approaching - not samples, but difference between them is transferred), subsampling, discrete Fourier cosine-transform, methods of motion compensation (information about direction of object element moves is formed), MPEG-1(2, 4) algorithms. By these methods of encoding it was succeeded to reduce television signal digital stream rate from 200 Mbit/s to 1,5 - 25 Mbit/s;

- voice message compression (telephony) is methods of prediction - CELP algorithm, vocoders (design of human vocal organs). If at telephony signals transmission by a standard PCM method in the digital channel R = 64 kbit/s, then the effective voice encoding methods allow to form the digital streams of 9,6 kbit/s (and even 4,8 kbit/s);

- musical message compression (sound broadcasting and television) is a method of the subband encoding (the signal spectrum is divided on bands, and the values of spectral density of different bands are quantized with the different quantization step) - MUSICAM algorithm. Higher class sound broadcasting signals and digitized records on disks by a PCM method require R = 700 kbit/s on one monophonic channel; the sound broadcasting signals effective encoding reduce rate to 100 kbit/s.

Last ten years there is rapid adoption of digital methods of television and sound broadcasting signals transmission. Main advantage is a high carrying capacity of the transmission systems, if to express it by the number of the programs.

At the compression of converted in digital signals continuous messages, it is possible to use the methods of discrete messages compression. However, at the continuous messages compression (video, photo, voice and music) the loss of information it admits or, more exactly, the exception of unimportant information it admits. It is possible because of man feature sensing of these messages types, nonidealness of his sense-organs. In this case the degree of compression is limited to the required quality of messages transmission only.

3 INFORMATION CHARACTERISTICS OF COMMUNICATION CHANNELS

3.1 Models of communication channels

Two models of communication channels are most distributed: a digital communication channel and a continuous communication channel.

The communication channel is called **digital** if it is intended for transmission of a digital signal. Such channel is characterized:

– by the rate of digital signal R (bit per second) which can be transmitted by a communication channel;

– by the number of signal levels in a communication channel M;

– by the symbol rate, symbol/s, which shows quantity of channel symbols, transmitted on a communication channel per 1 second, $B = R/\log_2 M$;

– by probabilities of symbol transitions $P(\hat{b}_j / b_k)$ for k = 0, 1, 2, … M – 1, j = 0, 1, 2, … M - 1, where b_k – symbol on an input of the channel, $\hat{b}_j$ – symbol on an output of the channel.

The majority of digital communication channels are binary (M = 2), and for them it is carried out $P(\hat{b}_0 / b_1) = P(\hat{b}_1 / b_0) = p$, where p – probability of a bit error in a communication channel. Such communication channel is called as a **binary symmetric communication channel** (BSC). It represent as the graph (figure 17). BSC is characterized by two parameters: B and p.

The digital communication channel always is under construction on the basis of the continuous channel. The communication channel is called **continuous** if it is intended for transmission of a continuous signal. The most widespread model of the continuous channel is a Gaussian communication channel. Consider, that in such communication channel acts additive white Gaussian noise (AWGN), i.e. the noise has Gaussian distribution of probabilities of instant values with zero average and a uniform spectrum in a passband of a communication channel. The model of a communication channel is shown on figure 18. – relationship between an output and an input is described by a ratio $z(t) = s(t) + n(t)$.

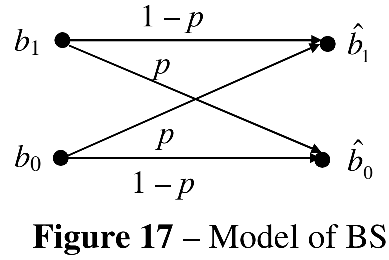

Figure 17 – Model of BSC

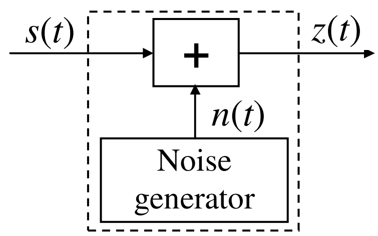

Figure 18 – Communication channel with AWGN

The Gaussian communication channel is described by the following parameters:

F_{ch} – passband of the channel;

P_s – average power of a signal on channel output;

N_0 – power noise density on channel output.

Consider, that noise takes place in a passband of a communication channel, and its average power on a channel output is defined

$$P_n = N_0\, F_{\text{ch}}.$$

3.2 Transmission rate of the information on a communication channel

Transmission rate of the information on a communication channel R_{ch} is defined as average quantity of the mutual information between input and output of the channel per 1 second. Unit of measure is bit/s. In the information theory consider, that errors or noise acting in a communication channel result in destruction of a part of the information.

Let's define transmission rate of the information in a digital communication channel. For this purpose we will take the concept of the mutual information of two sources of messages entered earlier. Let's consider here, that the output of one source is an input of a communication channel where the signal $b(t)$ operates, and the output of other source is an output of a communication channel where the signal $\hat{b}(t)$ operates. It is possible to write down

$$R_{\text{ch}} = \frac{I\left(\hat{A},\hat{B}\right)}{\grave{O}_{\text{av}}} = \frac{H(\hat{A}) - H\left(\hat{A}/\hat{B}\right)}{\grave{O}_{\text{av}}} = \frac{H\left(\hat{B}\right) - H\left(\hat{B}/\hat{A}\right)}{\grave{O}_{\text{av}}},$$

where $H(B)$ – entropy of a signal $b(t)$, describing average quantity of the information in one symbol on an input of a communication channel;

$H\left(B/\hat{B}\right)$ – conditional entropy of a signal $b(t)$ provided that the signal $\hat{b}(t)$ is known, characterizes losses of the information in a communication channel (unreliability of the channel);

$H\left(\hat{B}\right)$ – entropy of a signal $\hat{b}(t)$, describing average quantity of the information in one symbol on an output of a communication channel;

$H\left(\hat{B}/B\right)$ – conditional entropy of a signal $\hat{b}(t)$ provided that the signal $b(t)$ is known, describing quantity of the extraneous information (generated by errors) on an output a communication channel;

T_{av} – average duration of one symbol.

The similar approach is used for definition of transmission rate of the information in a continuous communication channel. On the input of a communication channel the signal $s(t)$ operates, and on the output of a communication channel the signal $z(t)$ operates. It is possible to write down

$$R_{\text{ch}} = \frac{h(S) - h(S/Z)}{T_s} = \frac{h(Z) - h(Z/S)}{T_s},$$

где $h(S)$ – differential entropy of a signal $s(t)$, describing average quantity of the information in one sample on an input of a communication channel;

$h(S/Z)$ – conditional differential entropy of a signal $s(t)$ provided that the signal $z(t)$ is known, characterizes losses of the information in a communication channel (unreliability of the channel);

$h(Z)$ – differential entropy of a signal $z(t)$, describing average quantity of the information in one sample on an output of a communication channel;

$h(Z/S)$ – conditional differential entropy of a signal $z(t)$ provided that the signal $s(t)$ is known, describing quantity of the extraneous information (generated by noise) on an output a communication channel;

T_s – sampling interval.

3.3 Capacity of binary symmetric communication channel

Communication channel capacity *C* is the greatest possible transmission rate of the information in communication channel at the set characteristics of a channel.

$$C = \max R_{\text{ch}}.$$

For calculation of capacity of BSC use next formula

$$C = \max \frac{H(B) - H\left(B/\hat{B}\right)}{T_{\text{av}}}.$$

Symbols have duration $T_{\text{av}} = 1/B$ (since a communication channel is binary than symbol rate coincides with a digital signal rate).

Further it is necessary to search for a maximum of values: $\max\left[H(\hat{A}) - H\left(\hat{A}/\hat{B}\right)\right]$. The maximum of the first term takes place, when symbols on an input of a liaison channel independent and equiprobable: $\max\left[H(B)\right] = 1$ binary unit.

Let's calculate the second term

$$H\left(B/\hat{B}\right) = -\sum_{k=1}^{M}\sum_{j=1}^{M} P\left(b_k, \hat{b}_j\right)\log_2 P\left(b_k/\hat{b}_j\right).$$

After substitution of $M = 2$ and values of probabilities from figure 17 we shall receive

$$H\left(B/B\right) = -p\log_2 p - (1-p)\log_2(1-p).$$

Let's remind, that this conditional entropy reflects losses of the information in a communication channel.

Final expression looks like

$$C = B\left[1 + p\log_2 p + (1-p)\log_2(1-p)\right].$$

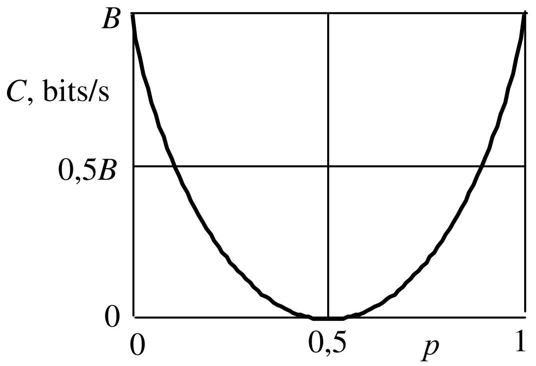

Figure 19 – Capacity of BSC

Special cases:

$p = 0$ – losses of the information are equal to zero and $C = R$;

$p = 1$ – all symbols in a communication channel are inverted, there is no losses of the information and $C = R$;

$p = 0{,}5$ – losses of the information

are equal to unit and $C = 0$;

3.4 Capacity of communication channel with AWGN

For calculation of a communication channel capacity with AWGN next formula is used

$$C = \max \frac{h(Z) - h(Z/S)}{T_s}.$$

Let's search for a maximum of values: $\max[h(Z) - h(Z/S)]$. The maximum of the first term takes place, when the signal $z(t)$ has Gaussian probability distribution. As $z(t) = s(t) + n(t)$, and noise $n(t)$ has Gaussian probability distribution, than $z(t)$ will have Gaussian distribution, if a signal $s(t)$ has Gaussian distribution. If it is correct, than

$$h(Z) = \log_2 \sqrt{2\pi e(P_s + P_n)}.$$

Conditional differential entropy of a signal $z(t)$ (provided that the signal $s(t)$ is known) is equal differential entropy of a Gaussian noise $n(t)$

$$h(Z/S) = h(N) = \log_2 \sqrt{2\pi e P_n}.$$

Sampling interval $T_s = 1/(2F_{\text{ch}})$, thus samples of signals are independent.

Let's substitute the received values:

$$C = 2F_{\text{ch}}\left[\log_2 \sqrt{2\pi e(P_s + P_n)} - \log_2 \sqrt{2\pi e P_n}\right].$$

After simple transformations we shall receive

$$C = F_{\text{ch}} \log_2\left(1 + \frac{P_s}{P_n}\right) = F_{\text{ch}} \log_2\left(1 + \frac{P_s}{N_0 F_{\text{ch}}}\right).$$

The ratio determining capacity of a communication channel with AWGN, is called as Shannon formula. From Shannon formula it is visible, that the basic resources of a communication channel are the passband of channel F_{ch} and power of a signal P_{s}. If $F_{\text{ch}} \to 0$ or $P_{\text{s}} \to 0$, than $C \to 0$.

If a passband of a communication channel tends to infinity, capacity of a Gaussian communication channel tends to final size

$$C_{F_{\text{ch}} \to \infty} = 1{,}443 \frac{P_s}{N_0}.$$

3.5 Efficiency coefficients of transmission system

For the estimation of as far as the basic resources of transmission system are effectively used, are used coefficient of frequency efficiency γ and coefficient of power efficiency β, determined by ratios:

$$\gamma = R_{\text{ch}}/F_{\text{ch}}; \qquad \beta = R_{\text{ch}}/(P_{\text{s}}/N_0).$$

Coefficient γ has dimension ((bit per second)/Hz) and shows, how many bits in a second are possible to transmit in a band of frequencies of 1 Hz with a considered method of transmission.

In transmission system at the fixed reliability of transmission the exchange of frequency efficiency for power efficiency and on the contrary is possible. Such exchange for ideal transmission system is expressed by a ratio

$$\beta = \frac{\gamma}{\left(2^{\gamma} - 1\right)}.$$

This ratio allows to fined limit value of one efficiency coefficient at fixed other and is called as **Shannon limit**. If as a measure of "perfection" of transmission system to accept a degree of approach of parameters γ and β to limiting on Shannon it is possible to fined, that the methods of coding of the channel developed for today (including modulation) are highly effective, and are not present significant reserves for increase of efficiency.

Except for the considered coefficients of efficiency β and γ the operating ratio of capacity of a communication channel is used also

$$\eta = R_{ch}/C.$$

3.6 Shannon theorem for a communication channel with noise

Importance of Shannon formula is connected with to well-known Shannon theorem for a communication channel with noise: If a message source rate is less than capacity of a communication channel ($R_s < C$) there is a way of the transmission, allowing to transfer all source messages as much as precisely.

Hence, noise in a communication channel doesn’t limit accuracy of message transfer, and rate of the information transfer on a channel.

THE LABORATORY WORK 2.1
INFORMATION CHARACTERISTICS OF MESSAGE SOURCES

1 Objectives

1.1 The studying the message sources information characteristics.

1.2 The acquiring the experience to perform the calculations of information characteristics of various sources.

2 Main positions

2.1 The information characteristics of discrete sources

The source of discrete messages produces the messages that consist of the separate signs and the quantity of this signs is finite. The signs may be symbols, letters, words, single phrases and so on.

Information quantity, $I(a_k)$, binary unit (or bit), in the sign (message) a_k, that has the probability of its appearance equals $P(a_k)$, is calculated on a formula

$$I(a_k) = -\log_2 P(a_k). \quad (1)$$

The source entropy, $H(A)$, is the average quantity information, that one sign keeps. If signs are independent, the source entropy can be calculated as the average value:

$$H(A) = -\sum_{k=1}^{M} P(a_k)\log_2 P(a_k), \quad (2)$$

where M – a size of source alphabet.

The entropy, as well as the information, is always nonnegative, and it achieves a maximal value, equals

$$H_{\max}(A) = \log_2 M , \quad (3)$$

if the signs are independent and equiprobable.

The message source *redundancy* is the source property to get out the information, using the greater quantity of signs, that it would be necessary. The availability of message source redundancy decreases the source entropy.

The *redundancy coefficient* of message source characterizes the relative decreasing of source entropy in compare with its maximal value:

$$\chi_{\mathrm{r}} = \frac{H_{\max}(A) - H(A)}{H_{\max}(A)}. \quad (4)$$

The *message source rate*, R_s, bit/s, is the average quantity of information, that the source gives out during 1 second. It is equals

$$R_{\mathrm{s}} = \frac{H(A)}{T_{\mathrm{sign}}}, \quad (5)$$

where T_{sign} – average time spent by a source on delivery with one sign.

2.2 The information characteristics of continuous message sources

Differential entropy, $h(B)$, of continuous source B is calculated on a formula:

$$h(B) = -\int_{-\infty}^{\infty} p(b)\log_2 p(b), \tag{6}$$

$p(b)$ is a probability density of the signal $b(t)$.

Differential entropy achieves its maximal value, if the probability density $p(b)$ is Gaussian, and it is equal

$$h(B) = \log_2 \sqrt{2\pi e\sigma^2}\ , \tag{7}$$

σ^2 is the variance of signal $b(t)$.

Epsilon entropy, $H_\varepsilon(B)$, bit/sample, is minimum average quantity of the mutual information between and $\hat{b}(t)$ in one sample at the given allowable error of approximate representation of a signal $b(t)$ by $\hat{b}(t)$: $\overline{\varepsilon^2(t)} \le \varepsilon_0^2$

The epsilon entropy is calculated on a formula:

$$H_\varepsilon(B) = \min\left\{h(B) - h\left(B/\hat{B}\right)\right\} = h(B) - \max h(E) \tag{8}$$

$h(E)$ is the differential entropy of error $\varepsilon(t)$.

The $h(E)$ will get the maximum value, if $\varepsilon(t)$ would have the Gaussian probability distribution, and the calculated formula for the epsilon entropy will be the same:

$$H_\varepsilon(B) = h(B) - \log_2 \sqrt{2\pi e\sigma^2}\ , \tag{9}$$

where σ^2 – variance of error $\varepsilon(t)$.

The *redundancy coefficient* of continuous message source is calculated

$$\chi_r = \frac{h_{max}(B) - h(B)}{h_{max}(B)}. \tag{10}$$

The *epsilon source rate*, R_s, of continuous message source is calculated on formula (5), and is necessary to put in it the value of epsilon entropy $H_\varepsilon(B)$, and T_{sign} would be equal to the sampling interval according the Kotelnikov theorem.

$$R_s = 2F_{max}h(B). \tag{11}$$

3 Questions

3.1 Give the determination of the information

3.2 How it is possible to calculate the information quantity that message contains?

3.3 Give the determinations of the source message information characteristics: the entropy, the differential entropy, the epsilon entropy, the source rate, and the source redundancy.

3.4 What parameters of discrete message source would be necessary known so as to calculate the entropy, the redundancy, and the rate?

3.5 What parameters of continuous message source would necessary is known so as to calculate the differential entropy, the epsilon entropy, the rate, and the redundancy?

4 Home task

4.1 Repeat the basic concepts of section "The source message information characteristics" using the summary and the textbooks [1, p. 101 – 106, 112 – 114, 124 – 129, 276 – 280]; [2, p. 276 - 280].

4.2 Calculate the information quantity I, that Your initials contain (three concrete letters), and compare it with the information quantity, that 15 bits contain (the information quantity in three source signs, when the signs are independent and equiprobable, and the volume of alphabet equals M = 32). Probabilities of letters are given in the appendix of À. If I will differ from 15 bits, explain this difference.

4.3 Prepare for discussing the key questions.

5 The order of study conduction

It is the calculating laboratory work, and is conducted in such order.

5.1 The discussion the key positions and calculation technique of source message information characteristics. The solution the typical examples, that are recommended to the study (point 6) or examples, that are prepared by the teacher (about 50 minutes).

5.2 The individual work with using individual cards (about 30 minutes).

Instruction: So far as the binary logarithm tables are absent, it is possible to use for the calculating the binary logarithms the mathematical rule of passing to the second basis of logarithm:

$$\log_2 z = \ln z / \ln 2 \approx 1{,}443 \ln z \,; \; \log_2 z = \lg z / \lg 2 \approx 3{,}32 \lg z \,.$$

6 The typical examples that are considered on the study

Example 1 Calculate the information quantity in the Ukrainian word of N = 8 letters. For calculations should be counted, that all the letters are of equal probability and independent, the number of letters M = 32.

The answer: I(word) = 40 bits.

Example 2 Calculate the binary message source redundancy, if the probability of one message $P(a_1) = 0{,}1$.

The answer: $\chi_r = 0{,}53$.

Example 3 Calculate the rate of message source; this source uses 3 independent signs, that have the probabilities: $P(a_1) =$ 0,1; $P(a_2) =$ 0,25; $P(a_3) =$? The duration of the 1-st message is equal 0,5 ms, of the 2-nd and 3-rd are equal 0,1 ms.

The answer: $H(A) =$ 1300 bit/s.

Example 4 Calculate the information characteristics of continuous message source: the epsilon entropy $H_{\varepsilon}(B)$; the redundancy coefficient χ_r; the epsilon source rate R_s.

Basic data**:** the continuous signal $b(t)$ has the Gaussian probability density and the maximum spectrum frequency $F_{\max}$ = 500 Hz; the ratio of the average signal power to the average error power ρ = 40 dB.

The answer: the epsilon entropy $H_{\varepsilon}(B)$= 6,64 bit/sample; the redundancy coefficient χ_r = 0; the epsilon source rate R_s = 6640 bits/s.

Literature

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2 **Панфілов І.П.,** Дирда В.Ю., Капацін А.В. Теорія електричного зв'язку: Підручник для студентів вищих навчальних закладів І та ІІ рівнів акредитації. – К.: Техніка,1998.

Appendix A

Reference tables for the calculations of information characteristics

Table A.1 – The letter probabilities distribution in the English texts

Letter	Probability	Letter	Probability	Letter	Probability	Letter	Probability
Blank	0.198	R	0.054	U	0.022	V	0.008
E	0.105	S	0.052	M	0.021	K	0.003
T	0.072	H	0.047	P	0.017	X	0.002
O	0.065	D	0.035	Y	0.012	J	0.001
A	0.063	L	0.029	W	0.012	Q	0.001
N	0.059	C	0.023	G	0.011	Z	0.001
I	0.055	F	0.022	B	0.010		

Table A.2 – The differential entropy calculating formulas

Probability distribution	Differential entropy h(B), bit/sample
$p(b)=\frac{1}{\sqrt{2\pi}\sigma_b}\exp\left(-\frac{b^2}{2\sigma_b^2}\right)$ – Gaussian	$\log_2(\sigma_b\sqrt{2\pi e})$
$p(b)=\frac{1}{\sqrt{2}\sigma_b}\exp\left(-\frac{\sqrt{2}\lvert b\rvert}{\sigma_b}\right)$ – two-sided exponential	$\log_2(\sigma_b e\sqrt{2})$
$p(b)=\begin{cases}0{,}5/b_{\max}, & \lvert b\rvert\le b_{\max}\\ 0, & \lvert b\rvert > b_{\max}\end{cases}$ – uniform	$\log_2(\sigma_b 2\sqrt{3})$

THE LABORATORY WORK 2.2
BASEBAND TELECOMMUNICATION SIGNALS SAMPLING

1. Objectives

1.1 Studying sampling of continuous signals and recovery of continuous signal from samples.
1.2 Analysing characteristics of discrete signals.

2 Main positions

2.1 Sampling of continuous signals.

The sampling of continuous signal $s(t)$ is representation it by its instant values (samples) $s(kT_s)$, where $k = \ldots, -1, 0, 1, 2, \ldots$; T_s – sampling interval. The sequence of samples represents with vertical lines in height $s(kT_\mathrm{d})$ (figure 1). Such sequence is named discrete signal $s_d(t)$.

In real devices signal sample $s(kT_s)$ is an pulse with amplitude $s(kT_s)$ and duration $\tau \le T_s$, beginning at the time moment kT_s. Usually $\tau \ll T_s$ (figure 2). The device that forms samples is called sampler. In a case $\tau < T_s$ sampler is the key closing a chain from a source to loading for time τ (figure 3).

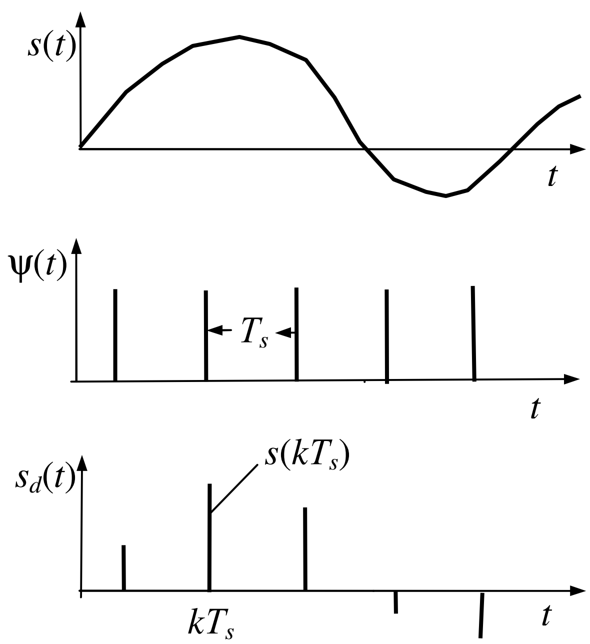


Figure 1 – Getting of discrete signal

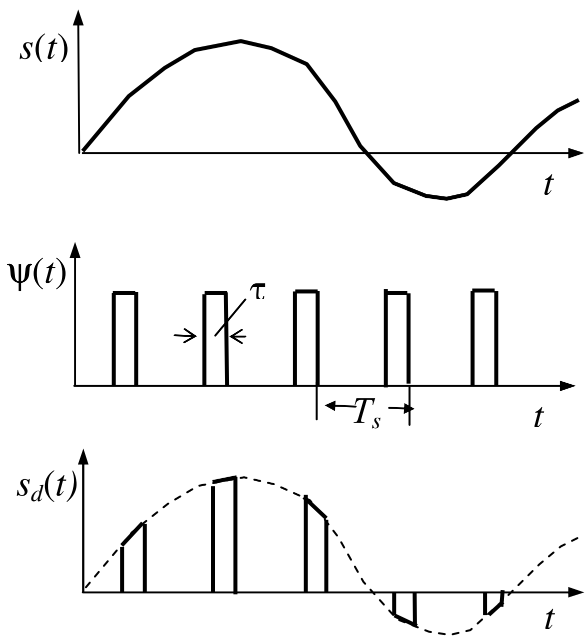
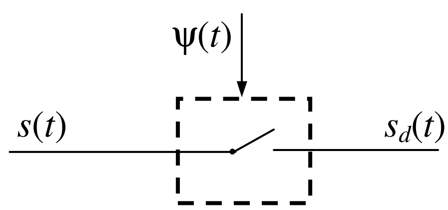


Figure 2 – Sampling process in real devices



Oscillograph

Analytical expression of a discrete signal $s_d(t)$:

$$s_d(t) = s(t)\cdot\psi(t) = s(t)\sum_{k=-\infty}^{\infty} h(t-kT_s)\,, \qquad (1)$$

where $\psi(t)$ – sequence sampling pulses that defines the time moments of sampling and their duration;

$h(t)$ – sampling pulse:

$$h(t) = \begin{cases} 1, & 0 \le t < \tau, \\ 0, & t < 0,\ t \ge \tau. \end{cases} \qquad (2)$$

2.2 Spectrum of a discrete signal.

The Fourier transform of right part expression (1) defines spectral density $S_d(j2\pi f)$ of discrete signal (corresponding mathematical calculations can be found in [1, p. 64–66])

$$S_d(j2\pi f) = \sum_{n=-\infty}^{\infty} a_n S(j2\pi(f - nf_s)) \quad -\infty < f < \infty, \qquad (3)$$

where $f_s = 1/T_s$ – sampling frequency;

$S(j2\pi f)$ – spectral density of a continuous signal $s(t)$;

$$a_n = \frac{\tau}{T_s}\cdot\frac{\sin(n\pi f_s\tau)}{n\pi f_s\tau} - \qquad (4)$$

decomposition factors $h(t)$ in Fourier series; as $\tau << T_s$ for small values n factors practically do not depend from n, that is $a_n = \tau / T_s$.

It is follows from (3), that the spectrum of a discrete signal is the sum of continuous signal spectrum $S(j2\pi f)$, repeating per f_s and decreasing with increase n according to expression (4).

For baseband telecommunication signals it is characteristic, that their spectrum adjoin zero frequency. On figure 4, *a* the amplitude spectrum of arbitrary form $S(f)$ with maximum frequency F_{max} is plotted. Further on figure 4 amplitude spectrum, which can take place at signal sampling, are represented:

Figure 4, *b* – sequence sampling pulses spectrum $S_\psi(f)$ $\psi(t)$, constructed on the basis of representation $\psi(t)$ in Fourier series:

$$\psi(t) = \sum_{n=0}^{\infty} a_n \cdot\cos(2\pi n f_s t);$$

Figure 4, *c* – discrete signal spectrum $S_d(f)$, if $f_s > 2F_{max}$;

Figure 4, *d* – spectrum $S_d(f)$, if $f_s = 2F_{max}$;

Figure 4, *e* – spectrum $S_d(f)$, if $f_s < 2F_{max}$.

2.3 Signals recovering on their samples.

According to **Kotelnikov theorem** (**the sampling theorem**) arbitrary signal with the band-limited spectrum can be recovered precisely to (interpolate) from its samples which are taken through interval $T_s \le 1/(2F_{\max})$, where $F_{\max}$ – the maximum frequency of signal spectrum.

It is easy to be convinced of Kotelnikov theorem justice, having considered figure 4, *c, d* and *e*. If $f_s \ge 2F_{\max}$ (figure 4, *c*, *d*) after giving of a discrete signal to an input ideal low pass filter (LPF) with cut frequency $F_{\max} \le F_{cut} \le f_s - F_{\max}$ on an output we will receive a signal with spectrum $S(f)$ (figure 4, *c*, *d*) that is the recovered continuous signal. On figures by dotted lines are shown the amplitude-frequency response (AFR) of ideal LPF with cut frequency $F_{cut} = F_{\max}$. If $f_s < 2F_{\max}$, that, apparently from figure 4, *e*, it is not impossible to recover spectrum $S(f)$ as spectrum aliasing takes place.

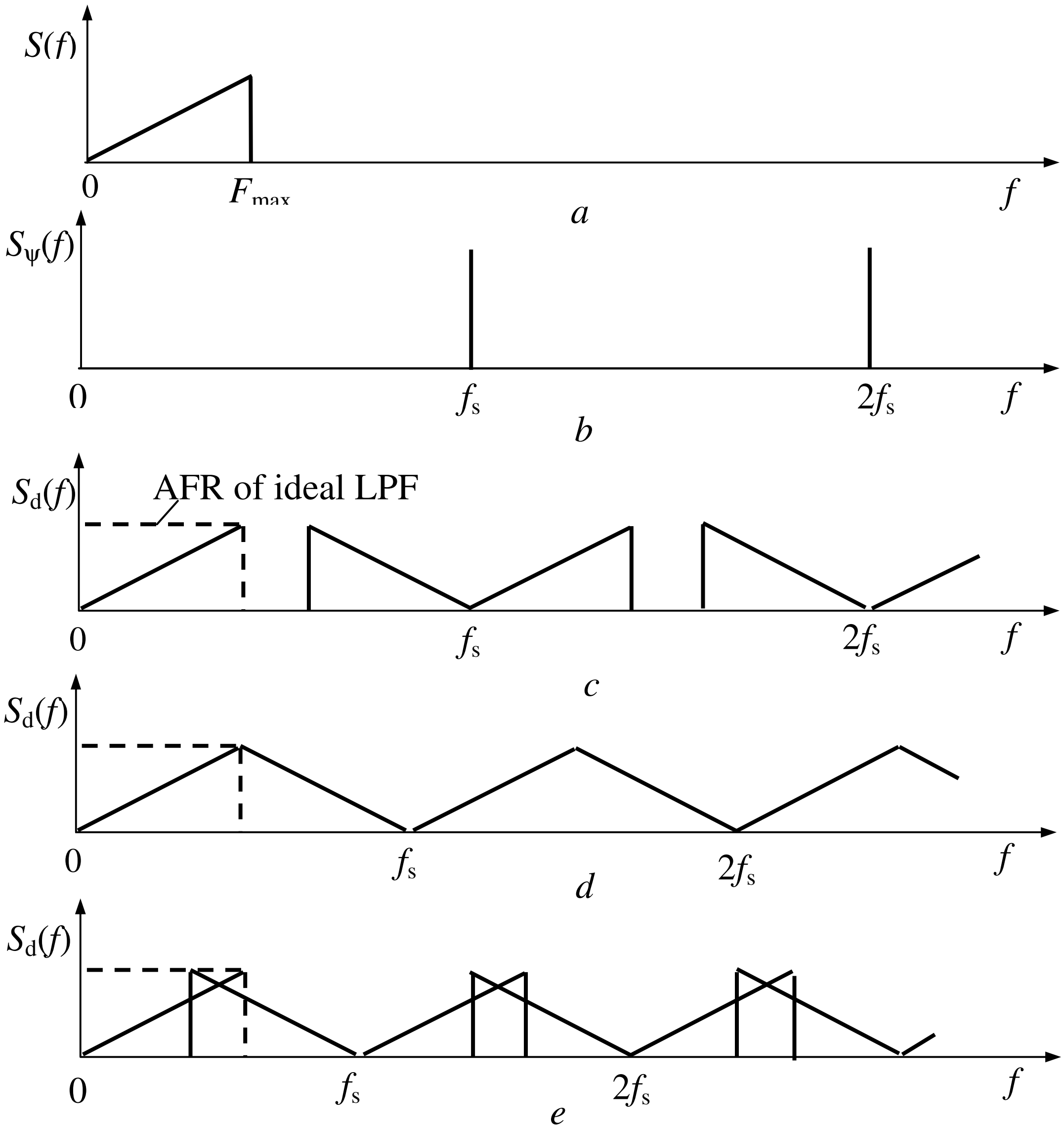


Figure 4 – The spectral diagrams sampling and continuous signals recovering processes illustration

Continuous signal recovering process on its samples can be also treated in time domain. If as signal recovering it is used ideal LPF with cut frequency F_{cut}, its impulse response (without a delay in the filter):

$$g(t) = \frac{\sin(2\pi F_{cut}\, t)}{2\pi F_{cut}\, t}.$$

As sample pulses short ($\tau << T_s$) (tend to δ-function) it is possible to consider, that response LPF to an pulse with amplitude $s(kT_s)$, supplied at the moment $t = kT_s$, looks like

$$s(kT_s) = \frac{\sin(2\pi F_{cut}\,(t - kT_s))}{2\pi F_{cut}\,(t - kT_s)}.$$

If you give to LPF input a signal $s_s(t)$, on its output we will receive the sum of responses

$$\hat{s}(t) = \sum_{k=-\infty}^{\infty} s(kT_д) \cdot \frac{\sin(2\pi F_{cut}(t - kT_s))}{2\pi F_{cut}(t - kT_s)}.$$

Let's compare this expression with **Kotelnikov series** that is mathematical expression of Kotelnikov theorem,

$$s(t) = \sum_{k=-\infty}^{\infty} s(kT_d) \cdot \frac{\sin(2\pi F_{max}(t - kT_s))}{2\pi F_{max}(t - kT_s)}.$$

If $F_{cut} = F_{max}$, $s(t) = \hat{s}(t)$, i.e. the continuous signal exact recovering takes place.

2.4 Errors at signals recovering on their sample

2.4.1 Real signals spectrum limitlessness. Real signals with strictly limited spectrum does not exist, as signals of finite duration have unlimited spectrum – at $f \to \infty$ spectrum decrease with limited speed. For real signals the spectrum maximum frequency F_{max} is defined from a condition, that components with frequencies $f > F_{max}$ are small (in a sense). In spectrum of real discrete signals there is an aliasing of spectrum, at least, the making sums (3) with indexes $n = 0$ and $n = 1$ (figure 5). We will assume, that for continuous signal's recovering the ideal LPF is used with cut frequency $F_{cut} = F_{max}$, its AFR is shown by a dotted line on figure 5.

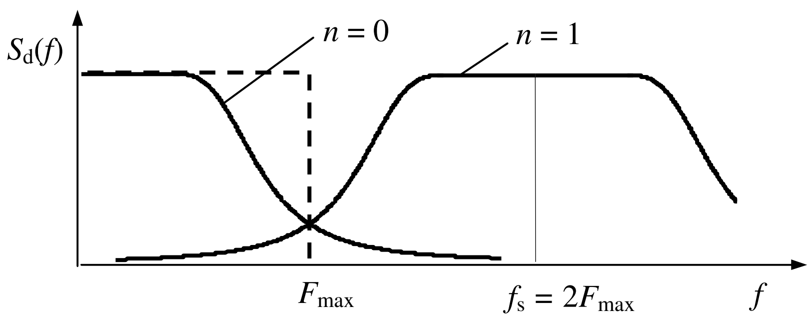


Figure 5 – Illustration of recovering errors because of continuous signal spectrum limitlessness

The recovered signal will have two making errors of recovering:

– linear distortions because of signal components $s(t)$ with frequencies $f > F_{max}$ are cutting off;

– spectrum components $S(f - f_s)$ with frequencies $f < F_{max}$ alias on a signal spectrum $s(t)$ (an aliasing error).

Taking this into account, value F_{max} and f_s define from a condition that the recovering error was enough small.

2.4.2 Difference real LPF characteristics from the ideal ones. Ideal LPF has the rectangular form AFR, and linear PFR. I.e., ideal LFF without distortions passes all signal spectrum components in band $f < F_{cut}$, and completely weakens components with frequencies $f > F_{cut}$. Real LPF are described by pass-band with boundary frequency F_{pb} and attenuation band with boundary frequency F_{ab} (figure 6).

If LFF is intended for recovery continuous signal with maximum frequency F_{max} from a discrete signal with frequency sampling f_s, it is necessary that $F_{pb} \geq F_{max}$ and $F_{ab} \leq f_s - F_{max}$. In case of real LPF there can be two making errors of recovering:

– because of inconstancy AFR and nonlinearity PFR in a pass-band the filter brings linear distortions in the recovering signal;

– because of insufficient easing in attenuation band LPF passes signal components $s_d(t)$ with frequencies $f > f_d - F_{max}$ which form an spectrum aliasing error.

Real LPF for recovering of continuous signals can be projected that the recovering error was enough small.

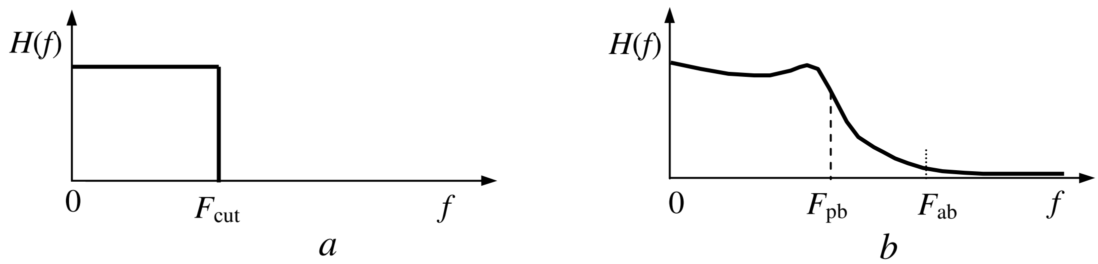


Figure 6 – AFR: *a* – ideal LPF; *b*– real LPF

3 Questions

3.1 Explain physical essence of a continuous signals sampling.

3.2 What purpose is continuous signals sampling carried out for?

3.3 Explain dependence of continuous and discrete signals spectrum.

3.4 Explain physical essence a signal on sample recovery process.

3.5 Formulate Kotelnikov theorem.

3.6 Write down Kotelnikov series for a signal with the band-limited spectrum.

3.7 In what basic differences AFR and PFR ideal and real LFF consist?

3.8 What are errors reasons which arise at signal restoration on samples?

4 Home task

4.1 Study section "Continuous signals sampling" under the lectures abstract and the literature [1, p. 59–67; 2, p. 64–69].

4.2 Signal $s(t) = A_1\sin(2\pi f_1 t) + A_2\sin(2\pi f_2 t) + A_3\sin(2\pi f_3 t)$ have sampling with frequency f_s. For the data specified in table 1 (according to number of your laboratory stand), represent a spectrum of signal $S(f)$ and a discrete signal spectrum $S_d(f)$, constructed in a range of frequencies $0 \le f \le 2f_s$.

Table 1 – Initial data to homework

Stand number	A_1, V	f_1, kHz	A_2, V	f_2, kHz	A_3, V	f_3, kHz	f_s, kHz
1	0	1	1	2	3	1,5	5
2	1	1	5	2,5	4	1,5	5,5
3	2	0,5	4	1,5	3,5	2,5	6
4	3	1	3	1,5	2,5	2,5	6,5
5	2,5	0,5	2	2,5	1	3	7
6	1,5	1	5	2,5	4	3	7,5
7	3,5	0,5	4	2	3	3	8
8	1	0,5	3	1	3,5	2	7

4.3 Calculate and draw impulse response ideal LPF with cut frequency $F_{cut} = f_s/2$ for values t, belonging to an interval $(-4T_s, 4T_s)$ (value f_s it is necessary to take from table 1).

4.4 Be prepared for discussion concerning section 3.

4.5 Study the laboratory model description (section 6).

5 Laboratory task

5.1 Familiarize with a virtual breadboard model. For this purpose start the program **1.0**, using an icon **TT(English)** on a desktop. Study the laboratory model scheme on the computer display, using section 6. Specify with the teacher the laboratory task-fulfilling plan.

5.2 Investigate sampling process in time and frequency domain. For this purpose set values A_1, f_1, A_2, f_2, A_3, f_3 and frequency f_s, given in a home task, to appoint input LPF from sampler and run the program. Sketch signals on source and sampler outputs oscillograms and the spectrograms in the report. Compare the spectrograms calculated and received on a laboratory model. Enter in the report comparison results. Increase sampling frequency on 1 kHz. Sketch in the report the signal spectrogram on an output sampler, make conclusions.

5.3 Investigate characteristics recovering LPF. Appoint influence on LPF δ-pulse and set LPF cut frequency value, which is given, in a home task for this purpose. Enter in the report the impulse response and AFR LPF. Compare impulse response LPF with calculated. Set sampling frequency twice smaller, enter in the report the impulse response and AFR LPF, make conclusions.

5.4 Investigate continuous signal recovery process a in time and frequency domain. Set parameters A_1, f_1, A_2, f_2, A_3, f_3, f_s and F_{cut}, given in a home task, give on LPF input a signal from sampler for this purpose. Compare oscillograms and spectrum on LPF output and on a source output, make conclusions.

Set LPF cut frequency smaller then frequency f_{max}, and then bigger then $f_s - f_{max}$. In both cases sketch oscillograms and spectrograms on output LPF, describe recovery errors character, explain the errors reasons.

6 Laboratory model description

Laboratory work is carried out on the computer in the HP VEE environment with virtual model use. The virtual model block diagram resulted on figure 7.

The laboratory model consists of: a continuous signal source $s(t) = A_1\sin(2\pi f_1 t) + A_2\sin(2\pi f_2 t) + A_3\sin(2\pi f_3 t)$, sampler, recovery LPF, the generator sampling pulses and the δ-pulses generator. It is possible to set the frequencies and amplitudes of harmonic waveform A_1, f_1, A_2, f_2, A_3, f_3, sampling frequency f_s and cut frequency LPF F_{cut} values.

The switch gives the chance to submit on an input recovering LPF a discrete signal $s_d(t)$ or δ-pulse. Time and spectral diagrams can be observed in three points of laboratory model: on a source output, on a LPF input and on a LPF.

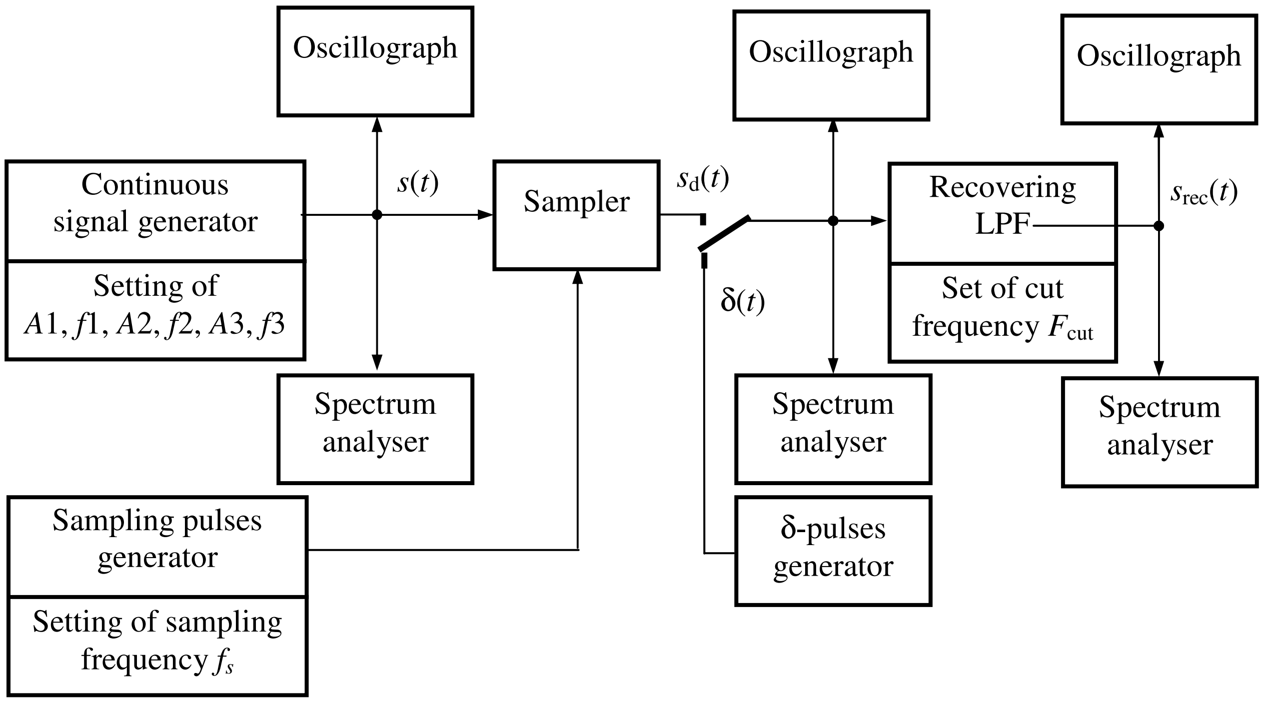


Figure 7 – Virtual model block diagram

7 Requirements to the report

6.1 Work purpose.
6.2 Homework performance results.
6.3 Laboratory task execution results.
6.4 Conclusions on each item of the laboratory task.

The literature

1. **Гоноровский И.С.** Радиотехнические цепи и сигналы. Учебник для Вузов. – М.: Радио и связь, 1986.

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THE LABORATORY WORK 2.3
RESEARCH OF ANALOGUE SIGNALS CODING BY PCM

1. Objectives

1.1 Studying the digital transmission of analog signals with PCM method.

1.2 Research the basic characteristics of the transmission system using the PCM method.

2 Main positions

2.1 The digital transmission methods

The digital transmission methods of analog signals are widely used in modern telecommunications: an analog signal is transformed in digital one, that is in the sequence of binary symbols; a digital signal is transmitted by the digital communication channel; after that an analog signal is recovered. Any digital transmission method is characterized by the rate of digital signal, R (bits/s), and by the transmission accuracy, that is by the signal/quantization noise ratio $\rho_{q.}$ The problem is usually set – to satisfy the requirement to the ρ_q when R has a minimum value. It was resulted in development of plenty digital transmission methods. The simplest method among them is pulse code modulation (PCM)[2].

2.2 The sampling of analog signals

At any digital transmission method the analog signal $b(t)$, first of all, is transformed in a digital signal, that is a sequence of samples $b(kT_s)$, taken with the sampling interval $T_s \le (1/2F_{\max})$, and $F_{\max}$ is the maximum frequency in the signal $b(t)$ spectrum. This transformation is called the signal sampling in the time, and device for its realization is called sampler. The sampling frequency must be not the less of doubled frequency $F_{\max}$:

$$f_s = 1/T_s \ge 2F_{\max}\,. \qquad (1)$$

In accordance with the Kotelnikov theorem, the execution of this relation guarantees the possibility of exact analog signal recovery from the samples. Such recovery is carried out in low-pass filter (LPF) cutoff frequency $F_{\max}$ (figure1).

Figure 1 – The analogue signal transmission by the samples

All digital transmission methods of analog signals differ in the ways of presentation the discrete signals in digital form. The transformer the samples in the

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[2] In spite of the presence the term “modulation”, these transmission methods are a relation neither to analog nor digital modulations

digital signal is called encoder of the digital transmission system (DTS), and transformer the digital signal in the samples is called the decoder of DTS.

2.3 The PCM methods

The peculiarity of these methods is that every sample is represented by the digital signal apart from other samples. These methods differ between itself by the used code. A scheme, that represents the sample transmission with PCM, is shown on a figure 2.

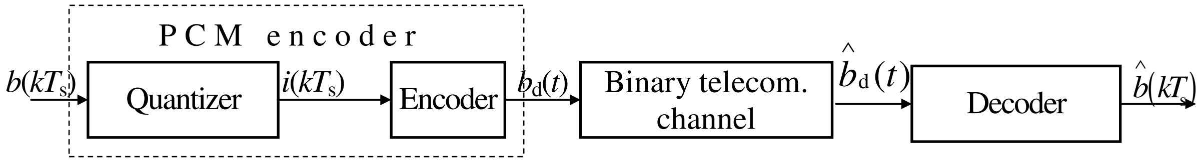

Figure 2 – Encoder and decoder PCM

The basic quantizer parameter is a number of quantization levels *L*. If the number of quantization is even, the values range of $b\,(-b_{\max};b_{\max})$ is broken up on $L-1$ intervals with the size equal

$$\Delta b = 2b_{\max}/(L-1), \quad (2)$$

that is called the quantization step. The breakdown with $L = 8$ is shown on the figure 3. The discrete values b_i correspond to the interval middles. The index i takes the values equal 0, ±1, ±2,…, ±0,5*L* – 1. Discrete values are determined as $b_i = i\cdot\Delta b$. During the quantization every sample $b(kT_s)$ is round off to the nearest discrete value b_i, and the integer number acts on the quantizer output. But the presentation the sample $b(kT_s)$ as a discrete value perform the error

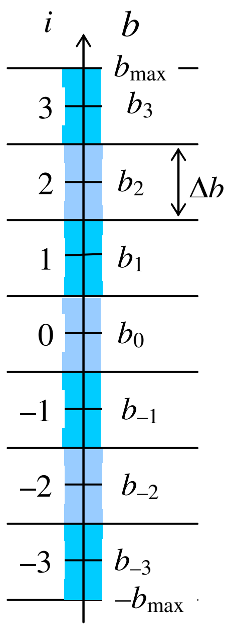

Figure 3 – The quantization explanation

$$\varepsilon_q(kT_s) = i(kT_s)\cdot\Delta b - b(kT_s), \quad (3)$$

and it is called as the quantization noise.

In the encoder, that is the part of PCM encoder (figure 2), the numbers $i(kT_s)$ are represented by the binary code, which is set. Code word length is equal

$$n = \log_2 L. \quad (4)$$

A digital signal rate at the encoder output is equal to

$$R = n\cdot f_s \quad (5)$$

The decoder forms the numbers $i(kT_s)$ from the digital signal $b_d(t)$. This numbers are used for the recovery the quantized sampling:

$$b_q(kT_s) = i(kT_s)\Delta b.$$

These are the recovered samples of analog signal:

$$\hat{b}(kT_s) = b_q(kT_s).$$

It is follows from the formula (3), that the samples are recovered with the errors $\varepsilon_q(kT_s)$. The average square error of quantization (the average power of quantization noise) is determined by the step of quantization:

$$\overline{\varepsilon_{\mathrm{q}}^2} = \frac{(\Delta b)^2}{12},$$

and signal/(quantization noise) ratio is equal:

$$\rho_{\mathrm{q}} = \frac{P_b}{\overline{\varepsilon_{\mathrm{q}}^2}} = \frac{3(L-1)^2}{K_A^2}, \qquad (6)$$

K_A is the amplitude coefficient of analog signal.

There are widely used PCM methods with the ununiform quantization: in area of large values of $|b|$ the quantization steps are large and vice versa. It is equivalent nonlinear transformation the samples with the following uniform quantization. Due to such transformation K_A decreases, and ρ_q increases, and *L* is constant, according to the formula (6). It is possible, if the value ρ_q wouldn't change, to decrease *L*, the code word length *n* and rate of digital signal *R* according to the formulas (4) and (5).

3 Questions

3.1 Explain the digital signal forming principle in the PCM method transmission system.

3.2 How is the sampling interval or the sampling frequency determined?

3.3 What is the quantization step and how it can be defined?

3.4 What code word length depends of in PCM systems?

3.5 Explain, what's noise of quantization? What is the reason of its origin?

3.6 How it is possible to increase the signal/(quantization noise) ratio in the transmission systems using the PCM methods?

4 Home task

4.1 Study the section "Digital methods of transfer of continuous messages", using the summary and literature [1, p. 242...252], [2, p. 262...270].

4.2 Represent the structure schemes of PCM encoder and decoder.

4.3 Calculate the characteristics of analog signal transmission system by the PCM method with uniform quantization. It is set: the sample frequency is equal to 8 kHz; the quantization level quantity $L_1 = 2^{N+2}$ (N is the brigade number); $L_2 = 2L_1$; $L_3 = 2L_2$. Calculate for three values of L: the quantization step Δb ($|b(t)|_{\max} = 1$ V^2); the code length n; the average quantization noise power $\overline{\varepsilon_{\mathrm{q}}^2}$ (the analog signal amplitude coefficient K_A = 2,85); the signal/(quantization noise) ratio ρ_q, dB; the digital signal rate R. Present the results of calculations like table 1. Analyze, how ρ_q and R will change if the code length would changes on unit.

Table 1 – The characteristics of the transmission system by the PCM method

f_s, kHz	L	n	R, kbits/s	P_b, V^2	Δb, V	$\overline{\varepsilon_q^2}$, V^2	ρ_q, dB
	L_1						
	L_2						
	L_3						

4.4 Be prepared for discussion concerning section 3.

5. Laboratory task

5.1 Familiarize with a virtual breadboard model. For this purpose start the program 2.10*a*, using an icon TT(English) on a desktop. Study the laboratory model scheme on the computer display, using section 6. Specify with the teacher the laboratory task-fulfilling plan.

5.2 Research the origin of quantization noise

Set the number L_1 (from a home task) of quantization levels. Present the results of fulfilling the program like table 2: write down the Δb and $b(kT_s)$, $i(kT_s)$, $b_q(kT_s)$ for k = 1, 2, 3 and 4. Calculate the values of $\varepsilon_q(kT_s)$, compare them with Δb, $\overline{\varepsilon_q^2}$ and explain a result. Repeat the task for L_2.

Table 2 – Analysis of quantization error

L	Δb	k	$b(kT_s)$, V	$i(kT_s)$	$b_q(kT_s)$, V	$\varepsilon_q(kT_s)$, V
		1				
		⋮				
		4				
		1				
		⋮				
		4				

5.3 Research the signal/(quantization noise) ratio

Set the quantization levels number L_1 (from a home task). Write down the results of the program fulfilling to the table like table.1: the values of Δb, $\overline{\varepsilon_q^2}$ and P_b. Calculate the experimental value of ρ_q. Compare the got values Δb, $\overline{\varepsilon_q^2}$ and ρ_q with the results of their calculations in a home task. Repeat the task for L_2 and L_3. On the fulfilling results build the graphs of signal/(quantization noise) and of digital signal rate as the functions of quantization levels number. Carry out the conclusions.

6 Laboratory model description

Laboratory work is fulfilled on a computer in the HP VEE environment with using the virtual model. Its structure scheme is shown on a figure 4. The virtual model is based on the schemes, which are shown on the figure1 and 2.

The analog signal generator forms the signal as a sum of a few harmonic waves. The signal duration is equal 2 ms, and maximal frequency of its spectrum $F_{\max} = 3{,}4$ kHz. An analog signal is normalized so, that $|b(t)|_{\max}$ = 1 V.

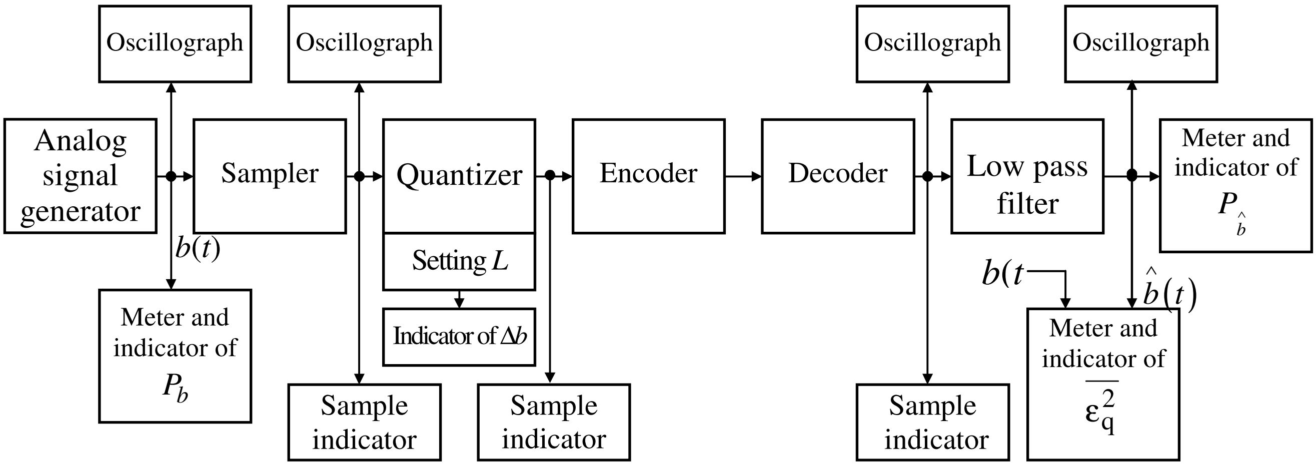

Figure 4 – Virtual model block diagram for research PCM method

7 Requirements to the report

7.1 The name of laboratory work.

7.2 The purpose of laboratory work.

7.3 The results of the home task.

7.4 The results of fulfilling the items 5.2 and 5.3 of laboratory task (tables, graphs).

7.5 The conclusions on every item of laboratory task, in which it necessary to give the analysis of the got results, such as the coincidence of theoretical and experimental data, etc.

7.6 The date, the signature of student, the visa of teacher with an estimation on a 100-mark scale.

Literature

1 **Теория** электрической связи: Учебник для вузов / А.Г. Зюко, Д.Д. Кловский, В.И. Коржик, М.В. Назаров; Под ред. Д.Д. Кловского. – М.: Радио и связь, 1998.

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THE LABORATORY WORK 2.4
COMMUNICATION CHANNELS INFORMATION CHARACTERISTICS

1 Objectives

1.1 The studying the communication channels information characteristics.

1.2 The acquiring the experience to perform the calculations of information characteristics of various channels.

2 Main positions

The transmission rate of the information on a communication channel is the mutual information between an input and an output of the channel per 1 s:

$$R_{\text{ch}} = \frac{I(\hat{A}, \hat{A})}{T_{\text{av}}}, \tag{1}$$

$I\left(B, \hat{B}\right)$ is the mutual information between an input and an output of the channel per one symbol;

B and $\hat{B}$ are the channel input and output messages;

T_{av} is the average time of one transmission symbol.

The message $\hat{b}(t)$ is distorted attitude to message $b(t)$ because of *noise* in an analog channel and *errors* in a digital communication channel, so the part of information is lost in a communication channel, and the mutual information $I\left(B, \hat{B}\right)$ is less than source entropy $H(B)$(the source is connected to the channel input), and information rate is less than source productivity.

The basic information characteristic of any communication channel is *channel capacity*, that determines the maximal possible transmission rate of information through this channel.

The *capacity* of the discrete symmetric communication channel *C*, bits/s, that assume the transmission passing the M symbols of identical duration, is calculated on a formula

$$C = B[\log_2 M + p \log_2 (p/(M-1)) + (1-p) \log_2 (1-p)], \tag{2}$$

B – is the symbol rate, Bods or syms/s;

p – is the symbol error probability in the communication channel.

The Gaussian channel *capacity*, bits/s, is calculated on a formula

$$C = F_{ch} \log_2 (1 + P_s / P_n), \tag{3}$$

F_{ch} is the communication channel passband;

P_s / P_n is the average powers of signal and noise ratio.

The Shannon theorem, that was formulated for the communication channel with noise is present, characterizes the potential possibilities of message transmission. The theorem asserts: if the source message rate R_s is less than the channel capacity *C*, that is $R_s < C_c$, there are a encoding method (the method of transformation the messages into the signal at the channel input) and a decoding method (the method of transformation the signal into the messages at the channel output), when the precision of message renewal would be high as someone want.

3 Questions

3.1 Give the determination of: the information rate; the channel capacity.

3.2 Explain the reasons of information losses in a discrete (continuous) communication channel.

3.3 What parameters of discrete symmetric communication channel and Gaussian communication channel would be known, so as to calculate their capacities?

3.4 What communication channel (or source) information characteristics are measured in bits/s or binary digits/s?

3.5 What are the conditions for the capacity became maximum (for the discrete symmetric channel and Gaussian channel)?

4 Home task

4.1 Repeat the basic concepts of section "Information characteristics of communication channels" using the summary and literature [1, p. 106-109; 114-122], [2, p. 281-284].

4.2 Prepare to the discussing the key questions.

5 The order of study conduction

It is the calculating laboratory work, and is conducted in such order.

5.1 The discussion the key positions and calculation technique of channel message information characteristics. The solution the typical examples, that are recommended to the study (point 6) or examples, that are prepared by the teacher (about 50 minutes).

5.2 The independent student work (ISW) with using individual cards (about 30 minutes).

6 The typical examples that are considered on the study

Example 1 Define the capacity of binary symmetric communication channel, if the symbol rate is equal B = 1000 symbs/s and probability of symbol errors $p = 10^{-3}$. How does symbol rate differ from the capacity of ideal channel (without errors)?

Answer: the capacity of this communication channel with errors is equal to 988,6 bits/s; the capacity of ideal communication channel is equal 1000 bits/s; the channel errors are resulted in the capacity decreasing of 11,4 bits/s (or 1,14 %).

Example 2 Calculate the channel capacity of a quaternary symmetric communication channel without errors, if the symbol rate is equal to 50 symbs/s.

Answer: C = 100 bits/s.

Example 3 Calculate the voice-frequency channel capacity (the cut-off frequencies are 0,3 kHz and 3,4 kHz), if at the channel output the average power of signal $P_s = 3{,}2 \cdot 10^{-5}\,\text{В}^2$, and the average power of noise $P_n = 8{,}7 \cdot 10^{-8}\,\text{В}^2$.

Answer: C = 2,64 bits/s.

Example 4 Is it possible to transmit, with a high quality, the 1-st class voice broadcast signal (the source epsilon-rate is equal to 240 kbits/s) through the baseband channel, that has the capacity equals 320 kbits/s, (see the table 18.1 [2])?

Answer: It is possible, because the source epsilon-rate (279 bits/s) is less than the channel capacity (320 bits/s).

Literature

1 **Теория** электрической связи: Учебник для вузов / А.Г. Зюко, Д.Д. Кловский, В.И. Коржик, М.В. Назаров; Под ред. Д.Д. Кловского. – М.: Радио и связь, 1998.

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INDIVIDUAL TASK № 2.1
CODING OF DISCRETE MESSAGE

Input data:

– the message made of a surname and a name of the student, carrying out the task;

– values of probabilities of letters in substantial English texts.

It is necessary:

1. Write out values of probabilities of letters and a blank, meeting in the given message from [5, LW 2.1a]. Execute normalization of these probabilities that their sum was equaled to unit. For this purpose it is necessary for probability of letters and a blank divide into the sum of probabilities.

2. Calculate entropy of the given message, considering, that the alphabet of a source is formed only from letters and a blank, meeting in a surname and a name, signs on the message are independent.

3. Carry out coding the given message by Shannon-Fano code or Huffman code (the variant of a code gets out on last digit of number of the student's record-book: odd – the Shannon-Fano code, even – the Huffman code).

4. Calculate average length of a code word of the letter and compare it to length of a code word of the letter at uniform coding.

5. Compare numerical values entropy and average duration of a code word of the letters. Explain in what ratio they can to be. Explain difference of these numerical values.

6. Calculate compression factor of the given message with the received code.

Methodical instructions to performance IT № 2.1

1. Information characteristics of message sources are in details described in [1, part 8], [2, part 18], [5, LW 2.1].

2. Effective coding by Shannon-Fano code is described in [1, p. 307–310], [4, p. 143], and effective coding by Huffman code is described in [4, p. 192].

3. Probabilities of letters in substantial English texts are resulted in “Help material to IT №2.1”.

4. Average length of code words is calculated as an average quantity of binary symbols in code words after coding by a nonuniform Shannon-Fano code or Huffman code.

5. Result the list of the literature in the executed task, and in the text to specify, from what reference (with the indication of number of subitem or numbers of pages) specific data for execute of the individual task are taken.

INDIVIDUAL TASK № 2.2
CODING OF CONTINUOUS MESSAGE

The continuous message of a source will be transformed to a baseband signal and then transferred by a communication channel method PCM with utilization of uniform quantization.

Input data:

– the maximal frequency in a spectrum of a baseband signal $F_{\max}$;
– average power of a baseband signal P_b;
– amplitude factor of a baseband signal K_A;
– the allowable ratio signal / quantization noise on output DAC $\rho_{q\ all}$.

The concrete numerical data get out on two last digits of number of the student's record-book from Table 3 – The initial data for IT №2.2 (p. 57).

It is necessary:

1. Make up and describe block diagram of ADC and DAC.
2. Define: the sampling frequency f_s; the sampling interval T_s; a number of quantization levels L; a length of a binary code n; a duration of a binary symbol T_b; a digital signal rate R.
3. Calculate the ratio signal / quantization noise ρ_q at chosen parameters of ADC.

Methodical instructions to performance IT № 2.2

1. The principle of construction ADC and DAC is described in the literature on telecommunication theory, for example, [2, part 17].
2. The technique of calculations ADC and DAC is resulted in [3, part 4] and in an electronic database of the chair.
3. Result the list of the literature in the executed task, and in the text to specify, from what reference (with the indication of number of subitem or numbers of pages) specific data for execute of the individual task are taken.

Literature

1. Стеклов В.К., Беркман Л.Н. Теорія електричного зв'язку: Підручник для студентів ВУЗів. За ред. В.К. Стеклова – К.: Техніка, 2006.

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5. **Методичні** вказівки до виконання лабораторних робіт з дисципліни “Теорія електричного зв’язку”. Частина 2 / Іващенко П.В. та ін. – Одеса: ОНАЗ ім. О.С. Попова, 2004.

Help material to individual tasks № 2.1 and № 2.2

Table A.1 – The letter probabilities distribution in the English texts

Letter	Probability	Letter	Probability	Letter	Probability	Letter	Probability
Blank	0.198	R	0.054	U	0.022	V	0.008
E	0.105	S	0.052	M	0.021	K	0.003
T	0.072	H	0.047	P	0.017	X	0.002
O	0.065	D	0.035	Y	0.012	J	0.001
A	0.063	L	0.029	W	0.012	Q	0.001
N	0.059	C	0.023	G	0.011	Z	0.001
I	0.055	F	0.022	B	0.010		

Huffman code [4, p. 192]

In case of Huffman coding algorithm signs of the source alphabet sign up in decreasing order their probabilities. If some signs have identical probabilities, they are placed in the any order. Then the probability tree as follows is under construction: choose two signs with the least probabilities and form the first branching of a tree. The chosen signs unite in the "intermediate" sign having probability, equal to the sum of probabilities of the chosen signs. Then among the staying signs (together with intermediate sign) again find two signs with the least probabilities and act the same as on a first step. This procedure carries out until will all signs on the source alphabet and intermediate signs will use up and the root of a tree which probability is equal 1 will be received.

«Movement» on a tree from a root to corresponding symbol carries out reception of code words. Passage through a branching means addition of a binary symbol in a code word: if through a branching "movement" upwards then unit is added and if downwards then zero is added.

Let's consider an example of coding of signs on the alphabet consisting of 7 signs. In tab. 2 signs are resulted and their probabilities are specified. On figure 1 construction of a probability tree is shown. For this purpose signs are located from top to down in decreasing order their probabilities. Procedure of construction of a probability tree, described is carried out above. The received code words are brought in tab. 2.

Calculation entropy of a considered source gives value

$$H(A) = -\sum_{k=1}^{M} P(a_k)\cdot \log_2 P(a_k) = 2{,}524 \text{ bit.}$$

Let's calculate average length of a code word

Table 3 – Huffman code

Sign a_k	$P(a_k)$	Code word
a_0	0,05	**10001**
a_1	0,3	**11**
a_2	0,2	**01**
a_3	0,1	**000**
a_4	0,06	**1001**
a_5	0,15	**101**
a_6	0,11	**011**
a_7	0,03	**10000**

$$\bar{n} = -\sum_{k=1}^{M} P(a_k) \cdot n_k = 2{,}72.$$

According to Shannon theorem of source coding the average length of a code word cannot be less source entropy. In our example

$$\bar{n} = 2{,}72 > H(A) = 2{,}524 \text{ bit}.$$

These values differ only on 7%.

If lengths of code words are identical and equal n the code is called uniform or primitive. The length of such code depends on size of the source alphabet:

$$n = \log_2 M = \log_2 8 = 3.$$

For an estimation of compression efficiency the parameter factor of compression is entered:

$$\mu = n/\bar{n} = 3/2{,}72 = 1{,}103.$$

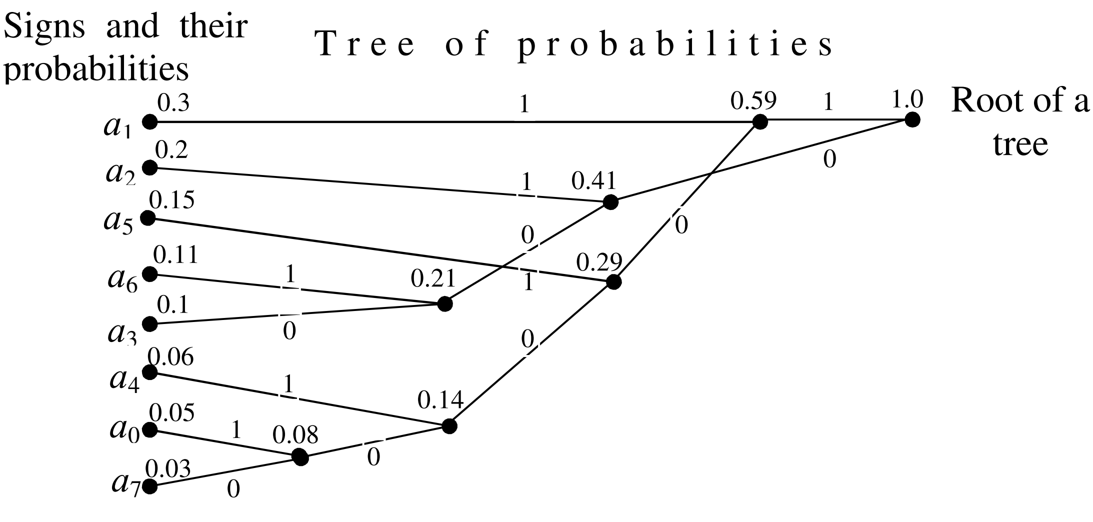

Figure 1 – Coding on Huffman algorithm

Calculation of ADC and DAC parameters
(it is carried out under the manual [3])

Input data (write out from table 3):
– the maximal frequency in a spectrum of a baseband signal F_{max};
– average power of a baseband signal P_b;
– amplitude factor of a baseband signal K_A;
– the allowable ratio signal / quantization noise on output DAC $\rho_{q\ all}$.

It is necessary to:
– make up and describe block diagram of ADC and DAC;
– define the sampling frequency f_s and the sampling interval T_s;

– define a number of quantization levels L, a length of a binary code n and a duration of a binary symbol T_b;

– calculate the ratio signal / quantization noise ρ_q at chosen parameters of ADC;

– calculate allowable probability of a symbol error of a p_{all} in a communication channel between ADC and DAC (on DAC output).

Calculating formulas

According to the Kotelnikov theorem [1, part 2.7; 2, part 2.4] the sampling frequency $f_s = 1/T_s$ should satisfy to a condition

$$f_s \geq 2F_{max}. \quad (1)$$

The sampling interval is value return sampling frequency

$$T_s = 1/f_s. \quad (2)$$

The noise immunity of transmission system of continuous messages is defined by the ratio signal / noise on an input recipient (in our case on output DAC)

$$\rho_{out} = P_b / \sigma_\varepsilon^2, \quad (3)$$

where σ_ε^2 – average power of noise on an input recipient.

In digital transmission system by method PCM power of a noise on output DAC is defined

$$\sigma_\varepsilon^2 = \overline{\varepsilon_q^2} + \overline{\varepsilon_{er}^2}, \quad (4)$$

where $\overline{\varepsilon_q^2}$ – average power of a quantization noise;

$\overline{\varepsilon_{er}^2}$ – average power of the noise caused by errors in a digital communication channel.

It is defined also the ratio signal / quantization noise in transmission system by method PCM

$$\rho_q = P_b / \overline{\varepsilon_q^2}. \quad (5)$$

Value $\rho_{кв}$ is defined at uniform quantization

$$\rho_q = 3(L-1)^2 / K_A^2. \quad (6)$$

Power of the noise, causing by errors in a digital communication channel, on output DAC is defined by a proportion [1, formula (8.14)]

$$\overline{\varepsilon_{er}^2} = p(\Delta b)^2 \sum_{i=1}^{n} 2^{2(i-1)} = p(\Delta b)^2 \frac{4^n - 1}{3}, \quad (7)$$

where p – probability of a bit error on input DAC;

Δb – step of quantization;

n – the length of binary code ADC connected to number of quantization levels

$$n = \log_2 L. \quad (8)$$

This ratio takes into account, that number of quantization levels L is the whole degree of number 2.

The baseband signal b (t) accepts values from b_{min} up to b_{max}. The interval (b_{min}, b_{max}) is subject to quantization. The quantization step is defined

$$\Delta b = (b_{max} - b_{min})/L. \quad (9)$$

If average value of a signal equal to zero, then $b_{min} = -b_{max}$. If value b_{max} is not given, it is defined from a proportion

$$b_{max} = K_A \sqrt{P_b}\ , \quad (10)$$

Duration of a binary symbol on output of ADC is defined

$$T_b = T_s / n. \quad (11)$$

Procedure of calculation

Block diagrams of ADC and DAC are in detail described in [1, part 8; 2, part 16]. It is necessary to add ADC circuits with LPF. LPF in real telecommunication systems is used for limitation of a baseband signal spectrum. It explains that at spectrum of baseband signals are slowly decreasing function. Value F_{max} is limiting frequency of a band, which is necessary for transmitting from a condition of achievement of the given quality of a baseband signal reproduction. Value F_{max} is defined by necessary legibility of speech, image sharpness, etc.

The increase in sampling frequency allows simplifying the input LPF of ADC and the output (interpolating) LPF of DAC. The LPF output of DAC recoveries a continuous signal on samples. The increase in sampling frequency results in reduction of binary symbol duration on output ADC. It causes undesirable spread of a frequency band of a continuous communication channel for transmitting of binary symbols. Usually parameters of the input LPF of ADC and the output LPF of DAC choose identical.

On figure 2 are given: $S(f)$ is a spectrum of the samples submitted by narrow pulses, $S_b(f)$ is a spectrum of a continuous signal $b(t)$, $A(f)$ is a performance attenuation of LPF. That LPF did not bring linear distortions in a continuous signal; limiting frequency of LPF pass band should satisfy to a condition

$$f_1 \geq F_{max}. \quad (12)$$

Limiting frequency of LPF stop band should satisfy to a condition

$$f_2 \leq (f_s - F_{max}). \quad (13)$$

It excludes aliasing $S_b(f)$ and $S_b(f - f_s)$ and provides attenuation components $S_b(f - f_s)$ with a recovering LPF.

That LPF were not too complex, the relation of limiting frequencies choose from a condition

$$f_2/f_1 = 1{,}3...1{,}4. \quad (14)$$

After substitution of proportions (12) and (13) in (14) it is possible to choose frequency of digitization. Then it is necessary to calculate a sampling interval.

Under the given allowable ratio signal / quantization noise $\rho_{q\ all}$ it is necessary to calculate allowable number of quantization levels L_{all} with the help of a proportion (6). Then choose $L \geq L_{all}$ and calculate n under the formula (8).

The ratio signal/noise given in decibels is necessary for presenting in times at carrying out of calculations

$$\rho = 10^{0,1\rho\ [dB]}. \tag{15}$$

Under the formula (6) it is necessary to calculate value ρq at chosen parameters ADC. Translate the designed value into decibels and compare with given $\rho_{q\ all}$.

Define allowable value of noise power caused by errors in a digital communication channel on the basis of proportions (3), (4) and (5), having accepted $\rho_{out} = \rho_{out\ all}$. The allowable error probability of a binary symbol on input DAC pall is defined from a proportion (7). The size of a quantization step is defined by the formula (9).

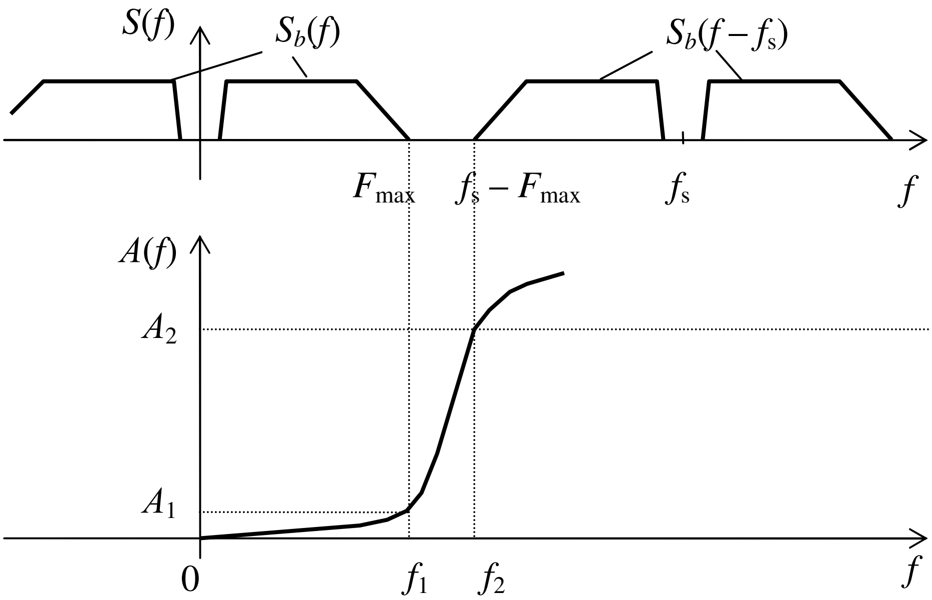

Figure 2 – The spectrum of samples and a filter amplitude response both ADC and DAC

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Table 3 – The input data for performance IT № 2.2

Number of a variant	Parameters of a message source			$\rho_{out\ all}$, dB	$\rho_{q\ all}$, dB
	P_b, V^2	K_A	F_{max}, kHz		
00	3,0	5	6,5	36	39
01	1,2	8	12	31	34
02	2,5	√3	2,4	38	41
03	0,1	5	6,5	42	45
04	0,3	5,5	8,0	42	45
05	0,5	√3	2,4	44	47
06	0,7	3	2,7	40	43
07	0,9	4	3,5	37	40
08	1,2	√3	5000	50	53
09	1,5	3,5	2,5	39	42
10	1,8	4,5	12	36	39
11	2,0	√3	3500	38	41
12	2,5	4,5	14	42	45
13	2,8	6,5	18	33	36
14	3,0	√3	800	44	47
15	0,2	7	12,5	39	42
16	0,4	8	15	37	40
17	0,6	√3	1,6	50	53
18	0,8	3,5	4,5	45	48
19	1,0	4,5	7,0	36	39
20	1,1	√3	0,8	38	41
21	1,3	5,5	7,5	42	45
22	1,4	6,5	9,5	37	40
23	1,6	√3	100	44	47
24	2,2	4,5	11	42	45
25	2,4	6,5	8,5	33	36
26	2,6	√3	0,1	50	53
27	1,9	3,5	2,5	45	48
28	0,1	3	2,7	46	49
29	0,3	√3	22	38	41
30	0,5	3,5	2,5	39	42
31	0,7	4,5	12	42	45
32	0,9	√3	110	44	47
33	1,1	4,5	14	36	39
34	1,3	7	12,5	39	42
35	1,5	√3	0,1	50	53
36	1,7	8	15	37	40
37	1,9	3,5	4,5	45	48
38	2,1	√3	180	38	41
39	2,3	4,5	7,0	36	39
40	2,5	5,5	7,5	42	45
41	2,7	√3	56	44	47
42	2,9	6,5	9,5	39	42
43	0,2	4,5	11	36	39
44	0,4	√3	44	50	53
45	0,6	6,5	8,5	33	36
46	3,5	0,8	2,5	45	48
47	1,0	√3	95	44	47

48	1,2	3	2,7	40	43
49	1,4	3,5	2,5	45	48

The ending of the table 3

Number of a variant	Parameters of a message source			$\rho_{out\ all}$, dB	$\rho_{q\ all}$, dB
	P_b, V^2	K_A	F_{max}, kHz		
50	1,6	√3	120	50	53
51	1,8	4,5	12	36	39
52	2,0	4,5	14	42	45
53	2,2	√3	144	38	41
54	2,4	3,5	22	39	42
55	2,6	3,5	1,5	45	48
56	2,8	√3	380	44	47
57	3,0	4,5	2,7	36	39
58	0,01	4,5	7,4	42	45
59	0,1	√3	500	50	53
60	0,02	5,5	16	35	38
61	0,2	5,5	12,5	41	44
62	0,03	√3	0,24	38	41
63	0,3	6,5	15	39	42
64	0,04	7,0	2,8	39	42
65	0,4	√3	12	44	47
66	0,05	7,5	3,3	32	35
67	0,5	8,0	4,4	31	34
68	0,06	√3	300	50	53
69	0,6	5	5,5	42	45
70	0,07	9,0	6,5	30	33
71	0,7	√3	9,6	38	41
72	0,08	3,5	4,3	45	48
73	0,8	4,5	7,5	42	45
74	0,09	√3	10,2	44	47
75	0,9	5,5	4,8	42	45
76	0,1	6,5	5,2	39	42
77	1,0	√3	1,5	50	53
78	0,12	7,5	3,6	38	41
79	1,2	8,0	10,0	37	40
80	0,15	√3	240	38	41
81	1,5	8,5	4,5	36	39
82	0,2	3	5,6	46	49
83	2,0	√3	480	44	47
84	0,25	4,5	11	36	39
85	2,5	5,5	14	35	38
86	0,3	√3	75	50	53
87	3,0	6,5	2,6	33	36
88	0,35	7,5	6,5	38	41
89	0,65	√3	68	38	41
90	0,4	6,5	16	39	42
91	0,7	5,5	6,3	42	45
92	0,45	√3	18	44	47
93	0,75	4	8,2	37	40
94	0,5	8	18	31	34
95	0,8	√3	72	50	53
96	0,55	4,5	6,4	42	45
97	0,85	6,0	12	40	43
98	0,6	√3	0,6	38	41
99	0,9	7,0	2,75	39	42

6 ENGLISH-RUSSIAN DICTIONARY

accumulator	накопитель
ADC (analog-to-digital convertor)	АЦП (аналого-цифровой преобразователь)
ADPCM (adaptive differential PCM)	АДИКМ (адаптивная дифференциальная ИКМ)
aliasing	наложение спектров
amplitude factor	коэффициент амплитуды
amplitude response (AR)	АЧХ (амплитудно-частотная характеристика)
AWGN (additive white Gaussian noise)	АБГШ (аддитивный белый гауссовский шум)
binary channel	двоичный канал
channel capacity	пропускная способность
code length	длина кода
communication channel	канал связи
compression of information	сжатие информации
conditional entropy	условная энтропия
continuous source	источник непрерывных сообщений
cut-off frequency	частота среза
DAC (digital-to-analog converter)	ЦАП (цифро-аналоговый преобразователь)
decoding method	метод декодирования
degree of compression	степень сжатия
differential entropy	дифференциальная энтропия
digital transmission	цифровая передача
discrete source	источник дискретных сообщений
DM (delta modulation)	ДМ (дельта модуляция)
effective coding	эффективное кодирование
encoding method	метод кодирования
epsilon entropy	эпсилон-энтропия
epsilon rate	эпсилон-производительность
equal-length code	равномерный код
frequency efficiency	частотная эффективность
Huffman code	код Хаффмана
information characteristics	информационные характеристики
information efficiency	информационная эффективность

information quantity	количество информации
instantaneous values	мгновенные значения
joint entropy	совместная энтропия
linear distortions	линейные искажения
low-pass filter (LPF)	фильтр нижних частот (ФНЧ)
message source	источник сообщений
mutual information	взаимная информация
non-uniform code	неравномерный код
power efficiency	энергетическая эффективность
predictor	предсказатель
primitive code	примитивный код
pulse code modulation (PCM)	импульсно-кодовая модуляция (ИКМ)
quantization noise	шум квантования
quantization step	шаг квантования
redundancy	избыточность
redundancy coefficient	коэффициент избыточности
sampling	дискретизация
Shannon theorem	теорема Шеннона
Shannon-Fano code	код Шеннона-Фано
signal/(quantization noise) ratio	отношение сигнал/шум квантования
slope overload	перегрузка по наклону
source entropy	энтропия источника
source rate	производительность источника
statistical code	статистический код
subdivision noise	шум дробления
symbol rate	скорость модуляции
time domain	временная область
transmission accuracy	точность передачи
uncorrelated messages	независимые сообщения
uniform quantization	равномерное квантование
variable-length code	неравномерный код

7 RUSSIAN-ENGLISH DICTIONARY

АБГШ (аддитивный белый гауссовский шум)	AWGN (additive white Gaussian noise)
АЦП (аналого-цифровой преобразователь)	ADC (analog-to-digital convertor)
АЧХ (амплитудно-частотная характеристика)	amplitude response (AR)
взаимная информация	mutual information
временная область	time domain
двоичный канал	binary channel
АДИКМ (адаптивная дифференциальная ИКМ)	ADPCM (adaptive differential PCM)
дискретизация	sampling
дифференциальная энтропия	differential entropy
длина кода	code length
ДМ (дельта модуляция)	DM (delta modulation)
избыточность	redundancy
импульсно-кодовая модуляция (ИКМ)	pulse code modulation (PCM)
информационная эффективность	information efficiency
информационные характеристики	information characteristics
источник дискретных сообщений	discrete source
источник непрерывных сообщений	continuous source
источник сообщений	message source
канал связи	communication channel
код Хаффмана	Huffman code
код Шеннона-Фано	Shannon-Fano code
количество информации	information quantity
коэффициент амплитуды	amplitude factor
коэффициент избыточности	redundancy coefficient
линейные искажения	linear distortions
мгновенные значения	instantaneous values
метод декодирования	decoding method
метод кодирования	encoding method
накопитель	accumulator
наложение спектров	aliasing
независимые сообщения	uncorrelated messages

неравномерный код	non-uniform, variable-length code
отношение сигнал/шум квантования	signal/(quantization noise) ratio
перегрузка по наклону	slope overload
предсказатель	predictor
примитивный код	primitive code
производительность источника	source rate
пропускная способность	channel capacity
равномерное квантование	uniform quantization
равномерный код	equal-length, fixed length code
сжатие информации	compression of information
скорость модуляции	symbol rate
совместная энтропия	joint entropy
статистический код	statistical code
степень сжатия	degree of compression
теорема Шеннона	Shannon theorem
точность передачи	transmission accuracy
условная энтропия	conditional entropy
фильтр нижних частот (ФНЧ)	low-pass filter (LPF)
ЦАП (цифро-аналоговый преобразователь)	DAC (digital-to-analog convertor)
цифровая передача	digital transmission
частота среза	cut-off frequency
частотная эффективность	frequency efficiency
шаг квантования	quantization step
шум дробления	subdivision noise
шум квантования	quantization noise
энергетическая эффективность	power efficiency
энтропия источника	source entropy
эпсилон-производительность	epsilon rate
эпсилон-энтропия	epsilon entropy
эффективное кодирование	effective coding

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Ivaschenko Peter Vasilyevich
Borschova Lesya Mikhailovna
Rozenvasser Denis Mikhailovich

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